PROGRESS IN MEASURING THE PRICE

AND QUANTITY OF CAPITAL

by

Erwin Diewert
University of British Columbia

and

Denis A. Lawrence
Tasman Asian Pacific

June 1999

Discussion Paper 99-17

DEPARTMENT OF ECONOMICS
THE UNIVERSITY OF BRITISH COLUMBIA
VANCOUVER, CANADA V6T 1Z1

http://web.arts.ubc.ca/econ/

Abstract

Griliches, Jorgenson and Hulten showed how to construct age specific user costs of capital or vintage rental prices that depended on the particular form of depreciation that was assumed. However, instead of assuming that the vintage rental prices represent the relative efficiencies of vintage specific assets and then using linear aggregation to aggregate up the vintage capitals into an aggregate capital, index number theory was used to aggregate up over vintages. In other words, each vintage capital was regarded as a separate input and superlative indexes were used to do the aggregation over vintages. Three different depreciation assumptions were considered: (1) declining balance or geometric depreciation; (2) one hoss shay and (3) straight line depreciation. It turns out that in the first two models, the sequence of vintage rental prices varies in strict proportion so Hicks' Aggregation Theorem applies and superlative indexes are not needed to aggregate over vintages. However, in the case of straight line depreciation, it is necessary to use a superlative index to perform the aggregation over vintages. The remainder of the paper illustrated what difference it made to Canadian productivity numbers.

Journal of Economic Literature Classification Numbers: C43, C82, D24, D92, E22, G12, M4, O47, O51, Q24

Keywords: capital, depreciation, Canadian productivity, user cost of capital, vintage capital stocks

PROGRESS IN MEASURING THE PRICE AND QUANTITY OF CAPITAL

By W. Erwin Diewert and Denis A. Lawrence[1]

1. Introduction

The fundamentals of capital measurement for production function and productivity estimation were laid out by Zvi Griliches (1963) over 35 years ago. This theory, which lays out the relationships between asset prices, rental prices, depreciation and the relative efficiencies of vintages of durable inputs, has been refined and extended by a large number of authors, including Jorgenson and Griliches (1967)(1972), Christensen and Jorgenson (1969)(1970), Jorgenson (1973)(1989)(1996), Diewert (1980), Hulten and Wykoff (1981a)(1981b)(1996), Hulten (1990)(1996) and Triplett (1996). Unfortunately, the United Nations (1993) System of National Accounts has not yet incorporated this well established theory into its production accounts, partly because the SNA regards interest as an income transfer rather than being a productive reward for postponing consumption and partly because capital gains are also regarded as being unproductive.[2] Thus from some points of view, there has been little official progress in measuring the price and quantity of capital in a form that would be suitable for production and productivity accounts.

However, the above paragraph presents a picture that is a bit too gloomy for two reasons:

- Several statistical agencies, starting with the U.S. Bureau of Labor Statistics[3], have introduced productivity accounts that are based on user costs[4];
- An international group of statistical agencies has set up a Working Group (the Canberra Group) under the direction of Derek Blades of the OECD whose mandate is to construct a handbook of capital measurement that would be used by national income accountants around

[1]Department of Economics, University of British Columbia, Vancouver, Canada and NBER and Tasman Asia Pacific, Canberra, Australia. The first author thanks the Canadian Donner Foundation for financial support and Michael Harper, Charles Hulten and Dale Jorgenson for helpful comments on an earlier draft. The first author would also like to dedicate this paper to Zvi Griliches who first introduced him to the difficult problems involved in economic measurement.

[2] To be fair to national income accountants, defining a coherent set of user costs or rental prices for capital stock components is not a trivial job. As we shall see later in this paper, there are many possible variants for user costs and it is difficult to select any single version. Diewert (1980; 470-486) discusses many of these variants.

[3] For descriptions of the BLS multifactor productivity accounts, see Bureau of Labor Statistics (1983) and Dean and Harper (1998).

[4] Statistics Canada and the Australian Bureau of Statistics also have multifactor productivity measurement programs.

the world. Hopefully, the user cost of capital will make its national income accounting debut in this document.

After delivering the above brief progress report, is there anything new that we can say in the remainder of the paper? We believe that there is. In the remainder of the paper, we flesh out a suggestion that dates back to Griliches:

> "Ideally, the available flow of services would be measured by machine-hours or machine-years. In a world of many different machines we would weight the different machine-hours by their respective rents. Such a measure would approximate most closely the flow of productive services from a given stock of capital and would be on par with man-hours as a measure of labor input."
> Zvi Griliches (1963), reprinted (1988; 127).

Following up on Griliches' suggestion, we will treat each vintage of a particular capital good as a *separate* vintage specific input into production and construct a *separate* rental price for that vintage. Then we will form a capital aggregate over vintages by using a superlative index number formula[5] that does not restrict a priori the substitution possibilities between the various vintages of that type of capital.[6] Thus instead of aggregating over vintages using an assumed pattern of relative efficiencies, we use the theory of exact index numbers to do the aggregation. However, as we shall see, the use of Hicks' (1946; 312-313) Aggregation Theorem (applied in the producer context) leads to the emergence of some familiar capital aggregates in the end.

In the following section, we lay out the basic relationships between depreciation (the decline in value of an asset due to age) and the asset and rental prices of each vintage of a durable input. We look at the relationships between each of these three *profiles* of prices (or depreciation amounts) as functions of age, assuming that we can observe a cross section of asset prices by age of asset. It turns out that any one of these profiles determines the other two profiles.

In sections 3,4 and 5, we specialize the general model of section 2 to work out the implications of three specific models of depreciation or relative efficiency that have been proposed in the literature. In section 3, we consider *the declining balance or geometric depreciation* model while in section 5, we consider the *straight line depreciation* model. In section 4, we consider *the one hoss shay model of depreciation* which assumes that the efficiency and hence rental price of each vintage of the capital good is constant over time (until the good is discarded as completely worn out after N periods). This model is sometimes known as the *gross capital stock* model. Note that these models all assume that the real rate of interest r is constant at any point in time.

[5] See Diewert (1976)(1978a) for material on superlative index number formulae.
[6] We implicitly assume that deterioration and depreciation of the various vintages do not depend on use; only on the age of the input.

The models derived in sections 3-5 imply different measures for the aggregate service flow of capital. Hence, the use of these different capital flow measures will lead to different measures of total factor productivity growth. In sections 6-8, we use Canadian data for the private business sector for the years 1962-1996 to construct alternative capital flows and productivity measures using the alternative capital concepts developed in sections 3-5. Thus we ask the question: does the use of these alternative capital measures *empirically matter* for the purpose of productivity measurement?[7] In sections 6-8, we also address some of the complications associated with the measurement of real interest rates when rates of inflation for asset prices differ.

Section 9 offers some concluding comments while the Data Appendix briefly describes and lists the Canadian data that we use.

2. The Relationship between Asset Prices, Depreciation and Rental Prices

Consider a new durable input that is purchased at the beginning of a period at the price P_0. At this same point in time, older vintages of this same input can be purchased at the price P_t for a unit of the asset that is t years old, for t=1,2,…. Generally speaking, these vintage asset prices decline as the age of the asset increases. This sequence of vintage asset prices at a particular point in time,

(1) $P_0 , P_1 ,\ldots, P_t ,\ldots$

is called the *asset price profile* of the durable input.

Depreciation for a unit of a new asset, D_0, is defined as the difference in the price of a new asset and an asset that is one year old, $P_0 - P_1$. In general, *depreciation* for an asset that is t years old is defined as

(2) $D_t = P_t - P_{t+1} \qquad ; t = 0,1,2,\ldots.$

Obviously, given the asset price profile, the profile of depreciation allowances, D_t , can be calculated using equations (2). Conversely, given the sequence of depreciation allowances, the asset price profile can be calculated using the following equations:

(3) $P_t = D_t + D_{t+1} + D_{t+2} + \ldots. \qquad ; t = 0,1,2,\ldots..$

In addition to the asset price sequence $\{P_t\}$ and the depreciation sequence $\{D_t\}$, there is a sequence of rental payments to the vintage assets or the sequence of *vintage user costs*, $\{U_t\}$,

[7] Thus our study is similar in some respects to the empirical investigation of alternative rental prices made by Harper, Berndt and Wood (1989).

that an asset of age t can earn during the current period, t=0,1,2,…. If the real interest rate in the current period is r, then economic theory suggests that the price of a new asset, P_0 , should be equal to the rental for a new asset, U_0, plus the discounted stream of rentals or user costs that older vintage assets can earn. In general, the price of an age t asset , P_t, should be approximately equal to a discounted stream of rental revenues that the asset can be expected to earn for the remaining periods of its useful life:

$$(4)\ P_t = U_t + (1+r)^{-1}U_{t+1} + (1+r)^{-2} U_{t+2} + \ldots \qquad ; t = 0,1,2,\ldots$$

Equations (4) can be manipulated (use the equations for t and t+1) to give us a formula for U_t in terms of the asset prices:

$$(5)\ P_t = U_t + (1+r)^{-1}P_{t+1} \qquad ; t = 0,1,2,\ldots$$

Equations (5) then yield the following formula for the user cost of a t year old asset:

$$(6)\ U_t = P_t - (1+r)^{-1}P_{t+1} \qquad ; t = 0,1,2,\ldots$$

The interpretation of (6) is clear: the net cost of buying an asset that is t years old and using it for one period and then selling it at the end of the period is equal to its purchase price P_t less the discounted end of the period price for the asset when it is one year older, $(1+r)^{-1}P_{t+1}$. User cost formulae similar to (6) date back to the economist Walras (1954; 269) and the early industrial engineer Church (1901; 907-908). In more recent times, user cost formulae adjusted for income taxes have been derived by Jorgenson (1963) (1989) and by Hall and Jorgenson (1967). A simple method for deriving these tax adjusted user costs may be found in Diewert (1980; 471) (1992; 194).

The above equations show that the sequence of vintage asset prices $\{P_t\}$, the sequence of vintage depreciation allowances $\{D_t\}$, and the sequence of vintage rental prices or user costs $\{U_t\}$, *cannot be specified independently*; given any one of these sequences, the other two sequences are completely determined.[8] This is an important point since capital stock researchers usually specify a pattern of depreciation rates and *these alternative depreciation assumptions completely determine the sequence of vintage specific rental prices which should be used as weights when aggregating across vintages to form an aggregate capital stock component.*

[8] This important point was recognized by Hulten (1990; 129) as the following quotation indicates: "One cannot select an efficiency pattern independently of the depreciation pattern and maintain the assumption of competitive equilibrium at the same time. And, one cannot arbitrarily select a depreciation pattern independently from the observed pattern of vintage asset prices P^t_s (suggesting a strategy for measuring depreciation and efficiency). Thus, for example, the practice of using a straight line efficiency pattern in the perpetual inventory equation in general commits the user to a non straight line pattern of economic depreciation." Hulten's efficiency pattern is our user cost profile.

In what follows, we consider three alternative patterns of depreciation: (a) declining balance or exponential depreciation (the amount of depreciation for each vintage is assumed to be a constant fraction of the depreciated asset value at the beginning of the period); (b) one hoss shay depreciation (or light bulb depreciation) where the efficiency of the asset is assumed to be constant until it reaches the end of its life when it completely collapses and (c) straight line depreciation where the amount of depreciation is assumed to be a constant amount for each vintage until the asset reaches the end of its life.

3. **The Declining Balance Depreciation Model**

In terms of the sequence of vintage asset prices, this model can be specified as follows:

(7) $P_t = (1-\delta)^t P_0$; $t = 1,2,\ldots.$

where δ is a positive number between 0 and 1 (the constant depreciation rate). Thus from (7), we see that the vintage asset price declines geometrically as the asset ages. If we substitute (7) into (2), we see that:

(8) $D_t = [1 - (1-\delta)](1-\delta)^t P_0 = \delta(1-\delta)^t P_0 = \delta P_t$; $t = 0,1,2, \ldots.$

i.e., depreciation for a t year old asset is equal to the constant depreciation rate δ times the vintage asset price at the start of the period, P_t. Note that the second equality in (8) tells us that D_t declines geometrically as t increases.

Substituting (7) into (6) yields the following sequence of vintage rental prices:

(9) $U_t = (1-\delta)^t P_0 - (1+r)^{-1}(1-\delta)^{t+1}P_0 = (1-\delta)^t(1+r)^{-1}[r + \delta] P_0$; $t = 0,1,2,\ldots$

Thus the rental price for a new asset is (set t = 0 in the above equation):

(10) $U_0 = (1+r)^{-1}[r + \delta] P_0$.

Now substitute (10) into (9) and we find that the rental price for a t year old asset is a geometrically declining fraction of the rental price for a new asset:

(11) $U_t = (1-\delta)^t U_0$; $t = 1,2,\ldots.$

The above equations imply that the vintage specific asset rental prices *vary in fixed proportion over time*. This means that we can apply Hicks' (1946; 312-313) Aggregation Theorem to aggregate the capital stock components across vintages.[9] If I_0 is the new investment in the asset in the current period and I_t is the vintage investment in the asset that occurred t periods ago for t =1,2,…., then the current period value of the particular capital stock component under consideration, aggregated over all vintages is:

$$(12)\ U_0I_0 + U_1I_1 + \ldots = U_0[I_0 + (1-\delta) I_1 + (1-\delta)^2 I_2 + \ldots].$$

Thus (12) gives us the value of capital services over all vintages of the capital stock component under consideration. It can be seen that this value flow can be decomposed into a price term U_0 which is the user cost for a new unit of the durable input, times an aggregated over vintages capital stock K defined as

$$(13)\ K = I_0 + (1-\delta) I_1 + (1-\delta)^2 I_2 + \ldots$$

This is the standard *net capital stock model* that has been used extensively by Jorgenson and his associates; see Jorgenson (1963) (1983) (1984) Jorgenson and Griliches (1967) (1972) and Christensen and Jorgenson (1969).

Note that in this model of depreciation, it is not necessary to use a superlative index number formula to aggregate over vintages in this model since its use would just reproduce the decomposition into price and quantity components that is on the right hand side of (12); i.e., in this model, Hicks' Aggregation Theorem makes the use of a superlative formula superfluous.

We turn now to the one hoss shay model of depreciation.

4. The Gross Capital Stock Model

In this model, it is assumed that the efficiency of the asset remains constant over its life of say N years and then the asset becomes worthless. This means that the rental price for the asset remains *constant* over its useful life; i.e., we make the following assumption:

$$(14)\ U_t = U_0 \quad \text{for } t = 1,2, \ldots,N-1 \quad \text{and} \quad U_t = 0 \quad \text{for } t = N,N+1,N+2,\ldots.$$

We need a formula for the user cost of a new unit of the asset, U_0. Substituting (14) into equation (4) when t = 0 yields:

$$\begin{aligned} & P_0 = U_0 + (1+r)^{-1}U_0 + (1+r)^{-2}U_0 + \ldots + (1+r)^{-N+1}U_0 \\ (15)\quad & \quad = U_0\,(1+r)\, r^{-1}[1 - (1+r)^{-N}]. \end{aligned}$$

[9] Hicks formulated his aggregation theorem in the context of consumer theory but his arguments can be adapted to the producer context; see Diewert (1978b)

Now use (15) to solve for U_0 in terms of P_0 :

(16) $U_0 = P_0\, r\, (1+r)^{-1} [1 - (1+r)^{-N}]^{-1}$.

The capital aggregate in this model is simply the sum of the current period investment I_0 plus the vintage investments going back N – 1 periods:

(17) $K = I_0 + I_1 + \ldots + I_{N-1}$.

The corresponding price for this capital aggregate is U_0 defined by (16). Because the rental price is constant across vintages, we can again apply Hicks' Aggregation Theorem to aggregate across vintages; i.e., we do not have to use a superlative index number formula to aggregate over vintages in this model since the user costs of the vintages will vary in strict proportion over time. This is the standard *gross capital stock model* that is used by the OECD and many other researchers. The only point that is not generally known is that there is a definite rental price that can be associated with this gross capital stock and the corresponding quantity aggregate is consistent with Hicks' Aggregation Theorem.

For comparison purposes, it may be useful to have explicit formulae for the profile of vintage asset prices P_t and the vintage depreciation amounts D_t . In terms of U_0, these formulae are:

(18) $P_t = U_0 (1+r)\, r^{-1}[1 - (1+r)^{-(N-t)}]$ for $t = 0,1,2,\ldots,N-1$ and
$P_t = 0$ for $t = N,N+1,\ldots$ and

(19) $D_t = U_0 (1+r)^{1-N+t}$ for $t = 0,1,2,\ldots,N-1$ and
$D_t = 0$ for $t = N,N+1,\ldots$.

Of course, P_t declines as t increases (for t less than N) but D_t *increases* as t increases (for t less than N), which is quite different from the pattern of depreciation in the declining balance model where depreciation *decreases* as t increases.

It is important to use the above gross capital stock user costs as price weights when aggregating over different components of a gross capital stock in order to form an aggregate flow of services that can be attributed to the capital stock in any period. Many researchers who construct gross capital stocks for productivity measurement purposes use formula (17) above to construct gross capital stock components but then when they construct an overall capital aggregate, *they use the stock prices P_0 as price weights instead of the user costs U_0 defined by (16).* This will typically lead to an aggregate capital stock which grows too slowly since structures (which usually grow more slowly than machinery and equipment components) are given an inappropriately large weight when stock prices are used in place of user costs as price weights; see Jorgenson and Griliches (1972) for additional material on this point.

We turn now to our final alternative model of depreciation.

5. The Straight Line Depreciation Model

In this model of depreciation, the depreciation for an asset which is t years old is set equal to a constant fraction of the value of a new asset P_0 over the life of the asset; i.e., we have

(20) $D_t = (1/N)\, P_0$ for $t = 0,1,2,\ldots, N-1$ and $D_t = 0$ for $t = N,N+1,N+2,\ldots.$

where N is the useful life of a new asset. Using (3) and (20), we can deduce that the sequence of vintage asset prices is

(21) $P_t = [1 - t/N]P_0$ for $t = 0,1,2,\ldots, N-1$ and $P_t = 0$ for $t = N,N+1,N+2,\ldots.$

Using (6) and (21), we can calculate the sequence of vintage user costs:

(22) $U_t = [1 - t/N]P_0 - (1+r)^{-1} [1 - (t+1)/N]P_0$

(23) $\quad = (1+r)^{-1}[r + N^{-1} - tN^{-1}r]P_0$ for $t = 0,1,\ldots,N-1$ and

$U_t = 0$ for $t = N,N+1,\ldots$

Recall that in the declining balance model, depreciation *decreased* as the asset aged (see (8) above) and in the gross capital stock model, depreciation *increased* as the asset aged (see (19) above). In the present model, depreciation is *constant* over the useful life of the asset. Also recall that in the declining balance model, the vintage asset prices *decreased* as the asset aged (see (7) above) and in the gross capital stock model, the vintage asset prices also *decreased* as the asset aged (see (18) above). In the present model, the vintage asset prices also *decrease* over the useful life of the asset (see (21) above). Finally, recall that in the declining balance model, the vintage rental prices *decreased* as the asset aged (see (11) above) and in the gross capital stock model, the vintage rental prices *remained constant* as the asset aged (see (14) above). In the present model, the vintage asset prices also *decrease* over the useful life of the asset (see (23) above); i.e., U_t decreases from $(1+r)^{-1}[r + (1/N)]P_0$ when $t = 0$ to $(1/N)P_0$ when $t = N-1$.

How can we empirically distinguish between the three depreciation models? We know of only three methods for doing this: (a) engineering studies; (b) regression models, which utilize profiles of used asset prices[10]; and (c) regression models where production functions or profit functions

[10] See Beidelman (1973)(1976), Hulten and Wykoff (1981a)(1981b) and Wykoff (1989) for studies of this type. An extensive literature review of the empirical literature on depreciation rate estimation can be found in Jorgenson (1996).

are estimated where vintage investments appear as independent inputs.[11] In practice, it is difficult to distinguish between the declining balance and straight line models of depreciation since their price and depreciation profiles are qualitatively similar.

We now encounter a problem with the straight line depreciation model that we did not encounter with our first two models: the rental prices of the vintage capital stock components *will no longer vary in strict proportion over time unless the real interest rate r is constant over time.* Thus in order to form a capital services aggregate over the different vintages of capital, we can no longer appeal to Hicks' Aggregation Theorem to form the aggregate using minimal assumptions on the degree of substitutability between the different vintages.

The aggregate value of capital services over vintages is:

$$(24)\quad U_0I_0 + U_1I_1 + \ldots + U_{N-1}I_{N-1} = (1+r)^{-1}[r + (1/N)]P_0\, I_0 + \ldots + (1/N)P_0\, I_{N-1}\, .$$

It can be seen that the price of a new unit of the capital stock, P_0 , is a common factor in all of the terms on the right hand side of (24); this follows from the fact that P_0 is a common factor in all of the user costs U_t defined by (23). Thus we could set the price of the aggregate equal to P_0 and define the corresponding capital services aggregate as the right hand side of (24) divided by P_0 . However, to justify this procedure, we have to assume that each vintage of the capital aggregate is a perfect substitute for every other vintage with efficiency weights proportional to the user costs of each vintage. The problem with this assumption is if the real interest rate is not constant, then we are implicitly assuming that efficiency factors are changing over time in accordance with real interest rate changes. This is a standard assumption in capital theory *but it is not necessary to make this restrictive assumption.* Instead, we can use standard index number theory and use a superlative index number formula (see Diewert (1976) (1978b)) to aggregate the N vintage capital stock components: in each period, the quantities are I_0, I_1,…, I_{N-1} and the corresponding prices are the user costs U_0, U_1,…, U_{N-1} defined by (23). If we use the Fisher (1922) Ideal index, then this formula is consistent with the vintage specific assets being perfect substitutes but the formula is also consistent with more flexible aggregator functions.

We conclude these theoretical sections of our paper by noting that there was no need to use an index number formula to aggregate over vintages in the first two depreciation models considered above since under the assumptions of these models, the vintage rental prices will vary in strict proportion over time. Thus if we did use an index number formula that satisfied the proportionality test, then the resulting aggregates would be the same as the aggregates that were exhibited in sections 3 and 4 above. Most models of depreciation do not have vintage rental prices that vary in strict proportion over time so those two models are rather special. More complicated (but more flexible) models of depreciation are considered in Hulten and Wykoff

[11] For examples of this type of study, see Epstein and Denny (1980), Pakes and Griliches (1984) and Nadiri and Prucha (1996).

(1981a).[12] The aggregation of the vintage capital stocks that correspond to these more complicated models of depreciation could also be accomplished using a superlative index number formula.

We turn now to an empirical illustration of the above aggregation procedures using Canadian data for the market sector of the economy for the years 1962-1996.

6. Construction of the Alternative Reproducible Capital Stocks for Canada

From the Data Appendix below, we can obtain beginning of the year net capital stocks for nonresidential structures, K_{NS}, and machinery and equipment, K_{ME}, in Canada for 1962 and 1997. We also have data on annual investments for these two capital stock components, I_{NS} and I_{ME}, for the years 1962-1996. Adapting equation (13) in section 3 above, it can be seen that if the declining balance model of depreciation is the correct one for Canada, then the 1997 beginning of the year capital stock for each of the above two components should be related to the corresponding 1962 stock and the annual investments as follows:

(25) $K^{1997} = (1-\delta)^{35} K^{1962} + (1-\delta)^{34} I^{1962} + (1-\delta)^{33} I^{1963} + \ldots + (1-\delta) I^{1995} + I^{1996}$

where δ is the constant geometric depreciation rate that applies to the capital stock component. Substituting the data listed in the Data Appendix into (25) for the two reproducible capital stock components yields an estimated depreciation rate of $\delta_{NS} = .058623$ for nonresidential structures and $\delta_{ME} = .15278$ for machinery and equipment. Once these depreciation rates have been determined, the year to year capital stocks can be constructed (starting at t = 1962) using the following equation:

(26) $K^{t+1} = (1-\delta) K^t + I^t$.

The resulting beginning of the year *declining balance* capital stock estimates for nonresidential construction may be found in the second column of Table 1 below. However, for machinery and equipment, when we compared the stocks generated by equation (26) to the net stocks tabled in the Data Appendix, we found that the two series started to diverge around 1991. Hence we used variants of equation (25) above to fit two separate geometric depreciation rates for machinery and equipment; the first rate applies to the 30 years 1962-91 and is $\delta_{ME} = .12172$ and the second rate applies to the 6 years 1991-1997 and is $\delta_{ME} = .16394$. Using these two depreciation rates in equation (26) led to the beginning of the year *declining balance* capital stock estimates for machinery and equipment that are found in the second column of Table 2 below.

[12] The Bureau of Labor Statistics (1983) has also adopted a more complicated hyperbolic formula to model depreciation,

We turn now to the construction of the capital stocks that correspond to the straight line depreciation assumption. Letting I^t be constant dollar investment in year t as usual, if the length of life is N years, then the beginning of year t *straight line* capital stock is equal to:

(27) $K^t = (1/N)[NI^{t-1} + (N-1)I^{t-2} + (N-2)I^{t-3} + \ldots + (1)I^{t-N}]$.

Our investment data begins at 1962. In order to obtain straight line capital stocks that start at the year 1962, we require investment data for the previous N years. We formed an approximation to this missing investment data by assuming that investment grew in the pre 1962 period at the same rate as the net capital stock grew in the 1962-1997 period. The net capital stock for nonresidential structures, K_{NS} in the Data Appendix, grew at the annual (geometric) rate of 1.033347 for the 1962-1997 period while the net capital stock for machinery and equipment, K_{ME}, grew at the annual (geometric) rate of 1.060053. Thus for a given length of life N say for machinery and equipment capital, we took the 1961 investment for machinery and equipment to be the unknown amount I_{ME}^{1961}, and then defined the investment for 1960 to be $I_{ME}^{1961}/1.060053$, the investment for 1959 to be $I_{ME}^{1961}/(1.060053)^2$, etc. We then substituted these values into (27) with t = 1962 and solved the resulting equation for I_{ME}^{1961}, assuming that K_{ME}^{1962} = \$17,983.7 billion dollars, the starting value taken from the net capital stock listed in the Data Appendix. We could construct the straight line capital stock for machinery and equipment using our assumed life N, the artificial pre 1962 investment data and the actual post 1962 investment data using formula (27). We then repeated this procedure for alternative values for N. We finally picked the N, which led to the straight line capital stock which most closely approximated the net capital stock listed in the Data Appendix. For machinery and equipment, the best fitting length of life N was 12 years while for nonresidential structures, the best length of life was 29 years. These straight line capital stocks are reported in column 3 of Tables 1 and 2.

Table 1. Alternative Capital Stocks for Nonresidential Structures in Canada

Year	*Declining Balance*	*Straight Line*	*Gross*
1962	30006.6	30006.6	50410.1
1963	30807.5	30828.3	51912.2
1964	31649.0	31685.7	53466.5
1965	32854.3	32902.7	55397.6
1966	34266.5	34330.7	57568.5
1967	36088.6	36176.4	60193.1
1968	37604.7	37732.6	62578.4
1969	39001.9	39176.3	64892.1
1970	40321.1	40544.3	67166.7
1971	41912.9	42183.7	69746.9
1972	43536.1	43858.8	72405.9
1973	45047.2	45425.4	75000.6
1974	46790.4	47223.2	77867.1
1975	48699.9	49190.6	80951.4

1976	51106.5	51660.7	84592.5
1977	53250.5	53883.7	88057.9
1978	55579.4	56297.8	91778.2
1979	57913.1	58724.9	95582.1
1980	60829.7	61740.6	100046.0
1981	64294.4	65321.5	105167.5
1982	68091.1	69260.8	110760.3
1983	70977.3	72319.3	115599.4
1984	73125.6	74642.4	119801.9
1985	75081.2	76753.7	123867.4
1986	77236.7	79039.4	128174.8
1987	78886.4	80797.0	132027.7
1988	80678.2	82660.8	136042.0
1989	83016.6	85037.6	140627.7
1990	85435.0	87473.4	145347.7
1991	87728.5	89763.4	149999.2
1992	89651.3	91656.7	154504.9
1993	90358.7	92291.9	157820.4
1994	91054.5	92842.7	160752.6
1995	92237.8	93820.6	163935.5
1996	93303.7	94640.8	166577.8
1997	94586.6	95649.3	169698.6

Table 2. Alternative Capital Stocks for Machinery and Equipment in Canada

Year	*Declining Balance*	*Straight Line*	*Gross*
1962	17983.7	17983.7	30017.3
1963	18162.7	17850.3	30606.6
1964	18522.0	17869.8	31291.0
1965	19291.3	18286.0	32316.0
1966	20494.5	19144.4	33748.5
1967	22229.6	20561.7	35732.0
1968	23845.6	21905.9	37672.9
1969	24966.0	22789.3	39171.7
1970	26331.1	23929.0	40900.1
1971	27615.1	25009.7	42552.9
1972	28876.9	26086.8	44169.5
1973	30361.5	27405.5	45981.9
1974	32785.0	29692.8	48722.4
1975	35687.4	32525.6	53247.5
1976	38628.9	35373.7	57962.8
1977	41530.2	38146.7	62542.3

1978	44060.1	40519.8	66575.8
1979	46904.7	43179.5	70553.8
1980	50746.0	46850.6	75782.5
1981	56096.6	52062.8	83287.1
1982	63204.7	59058.4	92819.3
1983	67262.2	63074.3	100081.1
1984	70606.0	66265.2	106988.9
1985	74318.5	69656.2	114296.2
1986	79447.3	74306.4	122352.0
1987	85489.7	79823.3	131171.7
1988	93219.6	87028.0	142022.1
1989	103385.1	96705.1	155931.1
1990	113970.6	106880.4	171515.8
1991	122239.5	114729.0	185449.6
1992	124460.9	121535.7	198159.9
1993	126893.6	127858.7	209468.7
1994	127816.8	132128.5	217258.1
1995	130582.7	137743.2	229226.9
1996	134305.4	143770.9	242825.8
1997	138467.1	149714.6	256698.2

Once the “best” length of lives N for nonresidential structures (29 years) and machinery and equipment (12 years) have been determined, these lives can be used (along with our pre 1962 artificial investment data and our post 1962 actual investment data) to construct the one hoss shay or gross capital stocks using the following formula:

(25) $K^t = I^{t-1} + I^{t-2} + I^{t-3} +\ldots+I^{t-N}$.

These gross capital stocks are reported in column 4 of Tables 1 and 2.

In the following sections, we use the above capital stock and investment information to construct alternative aggregate capital services measures along with total primary input and productivity measures for Canada.

6. Alternative Productivity Measures for Canada Using Declining Balance Depreciation

From the Data Appendix, we have estimates for the price and quantity of market sector output in Canada for the years 1962-1996, P_Y and Q_Y; for the price and quantity of market sector labour services, P_L and Q_L; for the price and quantity of business and agricultural land, P_{BAL} and K_{BAL}; and for the price and quantity of beginning of the year market sector inventory stocks, P_{IS} and Q_{IS}. We also have estimates of the operating surplus for the market sector, OS, which is equal to the value of output, P_YQ_Y , less the value of labour input, P_LQ_L. From the previous section, we have estimates of the beginning of the year declining balance capital stocks for nonresidential structures K_{NS} and for machinery and equipment K_{ME}. The corresponding prices, P_{NS} and P_{ME}, are listed in the Data Appendix. Thus we have assembled all of the ingredients that are necessary to form the declining balance user costs for each of our four durable inputs (nonresidential structures, machinery and equipment, land and inventories) that were defined by (10) in section 3 above. The only ingredient that is missing is an appropriate real interest rate, r.

For each year, we determined r by setting the operating surplus equal to the sum of the products of each stock times its user cost. This leads to a linear equation in r of the following form for each period:

(25) $$(1+r)OS = (r+\delta_{NS})P_{NS}K_{NS} + (r+\delta_{ME})P_{ME}K_{ME} + rP_{BAL}K_{BAL} + rP_{IS}K_{IS}.$$

Once the interest rate r has been determined for each period, then the declining balance user costs for each of the four assets can be calculated, which are of the following form:

(26) $$(r+\delta_{NS})P_{NS}/(1+r),\ \ (r+\delta_{ME})P_{ME}/(1+r),\ \ rP_{BAL}/(1+r),\ rP_{IS}/(1+r).$$

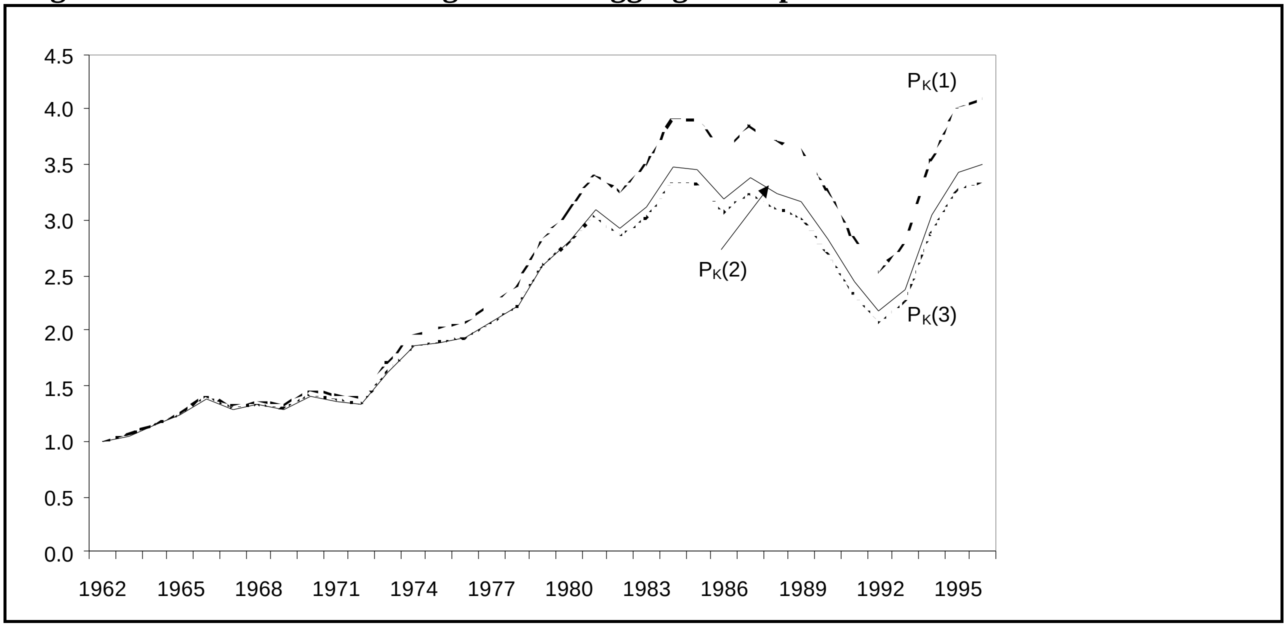
Finally, the above four user costs can be combined with the corresponding capital stock components, K_{NS}, K_{ME}, K_{BAL} and K_{IS}, using chain Fisher ideal indexes to form *declining balance capital price and quantity aggregates*, say $P_K(1)$ and $K(1)$.[13] The resulting aggregate price of capital services is graphed in Figure 1 below. We also combined the four rental prices and quantities of capital with the price and quantity of labour, P_L and Q_L, to form a primary input aggregate, $Q_X(1)$, (again using a chain Fisher ideal quantity index). Once this aggregate input quantity index $Q_X(1)$ was determined, we used our aggregate output index Q_Y along with the input index in order to define our first total factor productivity index, TFP(1):

(27) $$TFP(1) \equiv Q_Y/Q_X(1).$$

TFP(1) is graphed in Figure 2 below.

[13] For all of the capital models reported in this paper, the aggregate price of capital services P_K times the corresponding capital services aggregate K will equal the operating surplus OS.

Figure 1: Alternative Declining Balance Aggregate Capital Services Prices

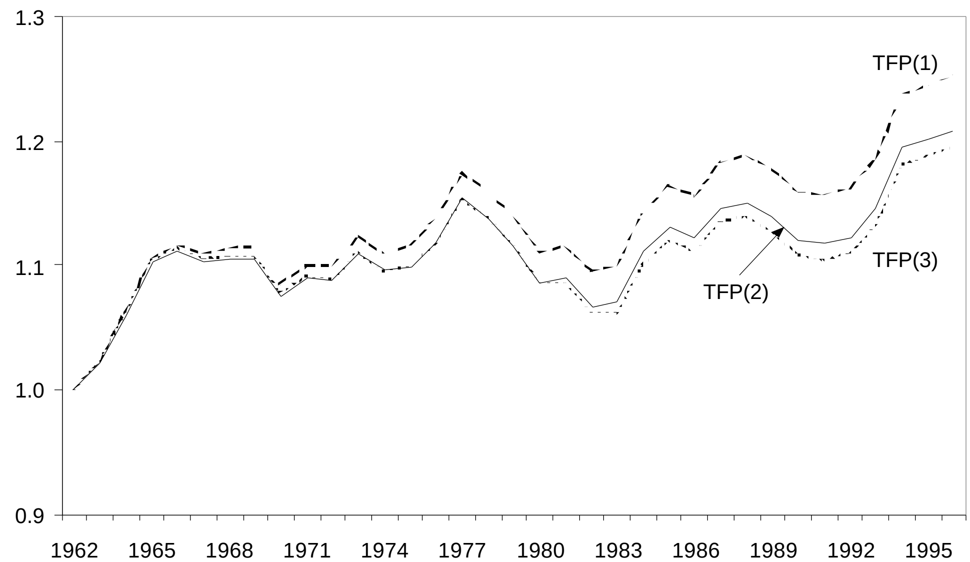


Since in many productivity studies (including ours), land is held fixed, it is often neglected as an input into production. However, even though the quantity of land is fixed, its price is not and so neglecting land can have a substantial effect on aggregate input growth. In order to determine this effect empirically, we recomputed the interest rate r for each period by using a new version of equation (29) above where the term $rP_{BAL}K_{BAL}$ on the right hand side of (29) was omitted. This omission of land has a substantial effect on the real interest rates: the average r increased from 5.933% to 7.808%. Once the new r's were determined, the three nonland user costs of the form (10) were computed. Then these three user costs were combined with the corresponding capital stock components, K_{NS}, K_{ME}, and K_{IS}, using chain Fisher ideal indexes to form new *declining balance capital price and quantity aggregates*, say $P_K(2)$ and K(2). The resulting aggregate price of capital services $P_K(2)$ is graphed in Figure 1. We also combine the three new rental prices and quantities of capital with the price and quantity of labour, P_L and Q_L, to form a new primary input aggregate, $Q_X(2)$, (again using a chain Fisher ideal quantity index). Once this aggregate input quantity index $Q_X(2)$ was determined, we used our aggregate output index Q_Y along with the input index in order to define our second total factor productivity index, TFP(2):

(28) $TFP(2) \equiv Q_Y/Q_X(2)$.

This second declining balance TFP measure (which omits land from the list of primary inputs) is graphed in Figure 2.

Figure 2: Alternative Declining Balance Productivity Measures



Many productivity studies also neglect the role of inventories as durable inputs into production. To determine the effects of omitting inventories on TFP in Canada, we recomputed the interest rate r for each period by using a new version of equation (29) above where both the land and inventory terms on the right hand side of (29) were omitted. This new omission of inventories has a further substantial effect on the real interest rates: the average r increased from 7.808% (with land omitted) to 10.067% (with land and inventories omitted). Once the new r's were determined, the two reproducible capital user costs of the form (10) were computed. Then these two user costs were combined with the corresponding capital stock components, K_{NS} and K_{ME}, using chain Fisher ideal indexes to form new *declining balance capital price and quantity aggregates*, say $P_K(3)$ and K(3). The resulting aggregate price of capital services $P_K(3)$ is graphed in Figure 1. We also combine the two new rental prices and quantities of capital with the price and quantity of labour, P_L and Q_L, to form a new primary input aggregate, $Q_X(3)$, (again using a chain Fisher ideal quantity index). Once this aggregate input quantity index $Q_X(3)$ was determined, we used our aggregate output index Q_Y along with the input index in order to define our second total factor productivity index, TFP(3):

(29) $TFP(3) \equiv Q_Y/Q_X(3)$.

This third declining balance TFP measure (which omits land and inventories from the list of primary inputs) is graphed in Figure 2.

Once a TFP^t measure has been determined for year t, we can define the *total factor productivity growth factor* ΔTFP^t and the corresponding *TFP growth rate* g^t for year t as follows:

(34) $\Delta TFP^t \equiv TFP^t / TFP^{t-1} \equiv (1 + g^t)$.

The TFP growth factors for the years 1963-1996 for each of the three declining balance TFP concepts that we have considered thus far are listed in the final table of the Data Appendix. However, the arithmetic averages of the three TFP growth rates for the 34 years 1963-1996, $g^t(1)$, $g^t(2)$, and $g^t(3)$, are listed in row 1 of Table 3 below.

Table 3. Averages of TFP Growth Rates for Declining Balance Models (%)

	g(1)	*g(2)*	*g(3)*	*g(4)*	*g(5)*	*g(6)*
1963-96	0.68	0.58	0.55	0.57	0.54	0.52
1963-73	1.08	0.97	0.97	0.98	0.96	0.96
1974-91	0.18	0.05	-0.01	0.03	0.00	-0.06
1992-96	1.63	1.60	1.62	1.61	1.60	1.62
average r or R	5.93	7.81	10.07	11.53	12.10	14.41
growth of K	3.89	4.36	4.51	4.42	4.52	4.63

Looking at column 1 of Table 3, it can be seen that TFP growth over the entire 34 years, 1963-1996 averaged .68% per year. However, this average growth rate conceals a considerable amount of variation within subperiods. For the 11 years before the first OPEC oil crisis, 1963-1973, the market sector of the Canadian economy delivered an average growth in TFP of 1.08% per year. During the following 18 years, 1974-1991, (which were characterized by high inflation, a growing government sector and higher tax levels), average TFP growth fell to .18% per year. After the recession in the early 1990's, TFP growth made a strong recovery, averaging 1.63% per year during the 5 years 1992-1996. The final two rows of Table 3 list the average interest rate that the capital model generated (which was 5.93% for our first declining balance model) along with the (geometric) average growth rate in real capital services (which was 3.89% per year for model 1).

When land is dropped as a factor of production (see column 2 of Table 3), the average interest rate increased to 7.89% and the average growth rate for capital services increased from 3.89% to 4.36% per year. This is to be expected: *excluding* land as an input (which does not grow over time) *increases* the overall rate of input growth and hence *decreases* productivity growth. Thus the average rate of TFP growth for Model 2 (which excluded land) has *decreased* to .58% per year from the Model 1 average rate of .68% per year—a drop of .1% per year.

Column 3 of Table 3 reports what happens when *both* inventories and land are dropped as factors of production. Since inventories have grown much more slowly than structures and machinery and equipment, *dropping* inventories further *increases* the average growth rate for real capital services, from 4.36% to 4.51% per year and further *decreases* the average TFP growth rate from .58% to .55% per year. However, the drop in the average TFP growth rate for the "lost" years, 1974-91, is even greater, from .05% to −.01% per year. Note that the average TFP growth rates for the recent "good" years, 1992-1996, do not differ much across the three declining balance capital models that we have considered thus far; the average annual TFP growth rates were 1.63%, 1.60% and 1.62% respectively.

The above 3 declining balance capital models were based on the theory outlined in sections 2 and 3 above. The analysis in these sections neglected *the inflation problem* or, more accurately, the above analysis implicitly assumed that asset inflation rates were identical across assets. We now want to relax this assumption and allow for differential inflation rates across assets.

The analysis in section 2 derived the relationships between vintage asset prices, depreciation and vintage user costs at one point in time, assuming no inflation. Hence the r which appeared in equations (4) to (6) can be interpreted as a real interest rate. We now want to generalize the fundamental user cost formula (6) to allow for asset inflation. We shall now use the superscript t to denote the time period and the subscript s to denote the vintage or age of the asset under consideration. Thus s = 0,1,2,… means that the asset is new (0 years old), 1 year old, 2 years old, etc. Let P_s^t denote the beginning of year t price of a capital stock component that is s years old and let R^t be the year t nominal interest rate. Then the year t *inflation adjusted user cost* for an s year old capital stock component, U_s^t, is defined as the beginning of year t purchase cost P_s^t less the discounted value of the asset one year later, P_{s+1}^{t+1}:

(35) $U_s^t \equiv P_s^t - (1+R^t)^{-1} P_{s+1}^{t+1}$; s = 0,1,2,…

We now make the simplifying assumption that the year t+1 profile of vintage asset prices P_s^{t+1} is equal to the year t profile P_s^t times one plus the year t inflation rate for a new asset, $(1 + i^t)$; ie, we assume that:

(36) $P_s^{t+1} = P_s^t (1 + i^t)$

where the year t *new asset inflation rate* i^t is defined as

(37) $1 + i^t \equiv P_0^{t+1}/P_0^t$.

Substituting (36) into (35) leads to the following formula for the *period t inflation adjusted user cost of an s year old asset*:

(38) $U_s^t = P_s^t - (1+R^t)^{-1}P_{s+1}^t (1+i^t)$.

The new user cost formula (38) reduces to our old formula (6) if the year t nominal interest rate R^t is related to the year t real rate r^t by the following Fisher effect equation:

(39) $1+R^t = (1+r^t)(1+i^t)$.

Substitution of (39) into (38) yields our old user cost formula (6) using our new notation. Thus is all asset inflation rates are assumed to be the same, our new user cost formula (38) reduces to our old formula (6). However, in reality, inflation rates differ markedly across assets. Hence, from the viewpoint of evaluating the ex post performance of a business (or of the entire market sector), it is useful to take ex post asset inflation rates into account.[14] If a business invests in an asset that has an above normal appreciation, then these asset capital gains should be counted as an intertemporally productive transfer of resources from the beginning of the accounting period to the end; i.e., the capital gains that were made on the asset should be offset against other asset costs. Thus in the remainder of this section, we use the inflation adjusted user costs defined by (38) in place of our earlier no capital gains user costs of the form (6).

The profile of year t vintage asset prices in the declining balance model of depreciation will still have the form given by (7). Using our new notation, (7) may be rewritten as:

$$(40)\quad P_s^{\,t} = (1-\delta)^s P_0^{\,t} \quad ; \quad s = 0,1,2,\ldots$$

Substituting (40) into (38) yields the following formula for the year t sequence of vintage inflation adjusted user costs:

$$\begin{aligned}(41)\quad U_s^{\,t} &\equiv (1-\delta)^s P_0^{\,t} - (1+R^t)^{-1}(1-\delta)^{s+1} P_0^{\,t}(1+i^t) \\ &= (1-\delta)^s (1+R^t)^{-1}\,[R^t - i^t + \delta(1+i^t)]P_0^{\,t} \\ &= (1-\delta)^s U_0^{\,t} \qquad ; \quad s = 0,1,2,\ldots\end{aligned}$$

where the year t declining balance inflation adjusted user cost for a new asset is defined as

$$\begin{aligned}(42)\quad U_0^{\,t} &\equiv P_0^{\,t} - (1+R^t)^{-1}(1-\delta)P_0^{\,t}(1+i^t) \\ &= (1+R^t)^{-1}\,[R^t - i^t + \delta(1+i^t)]P_0^{\,t}\,.\end{aligned}$$

Equations (41) show that all of the period t vintage user costs, $U_0^{\,t}$, $U_1^{\,t}$, $U_2^{\,t}$,…, will vary in strict proportion to the period t user cost for a new asset, $U_0^{\,t}$, and hence we can still apply Hicks' Aggregation Theorem to aggregate over vintage capital stock components. The capital stock aggregates that we used in Models 1-3 above, $K_{NS}^{\,t}$, $K_{ME}^{\,t}$, $K_{BAL}^{\,t}$ and $K_{IS}^{\,t}$, can still be used in our new Models 4-6 that allow for differential inflation rates. The only change is that the old user costs defined by (30) are now replaced by inflation adjusted user costs of the form given by (42) for each of our four capital stock components.

[14] From other points of view, ex post user costs of the form defined by (38) may not be appropriate. For example, if we are attempting to model producer supply or input demand functions, then producers have to form *expectations* about future asset prices; ie, *expected* asset inflation rates should be used in user cost formulae in this situation rather than *actual* ex post inflation rates.

Model 4 is an inflation adjusted counterpart to Model 1. Recall that we used equation (29) to solve for the real interest rate r for each year. The Model 4 counterpart to (29) is the following equation, which determines the nominal interest rate R for a given year:

$$(43)\ (1+R)OS = (R-i_{NS}+\delta_{NS}[1+i_{NS}])P_{NS}K_{NS} + (R-i_{ME}+\delta_{ME}[1+i_{ME}])P_{ME}K_{ME} + (R-i_{BAL})P_{BAL}K_{BAL} + (R-i_{IS})P_{IS}K_{IS}.$$

Once the nominal interest rates R^t have been determined for each year, then the declining balance user costs for each of the four assets can be calculated, which are of the form defined by (42).[15] The above four user costs can be combined with the corresponding capital stock components, K_{NS}, K_{ME}, K_{BAL} and K_{IS}, using chain Fisher ideal indexes to form *inflation adjusted declining balance capital price and quantity aggregates*, say $P_K(4)$ and $K(4)$. The resulting aggregate price of capital services $P_K(4)$ is graphed in Figure 3 below, along with $P_K(5)$ (where land is dropped as an input) and $P_K(6)$ (where both land and inventories are dropped as inputs). We also combined the four rental prices and quantities of capital with the price and quantity of labour, P_L and Q_L, to form the primary input aggregate, $Q_X(4)$, (again using a chain Fisher ideal quantity index). Once this aggregate input quantity index $Q_X(4)$ was determined, we used our aggregate output index Q_Y along with the input index in order to define the corresponding total factor productivity index, TFP(4):

$$(44)\ TFP(4) \equiv Q_Y/Q_X(4).$$

TFP(4) is graphed in Figure 4 below, along with TFP(5) and TFP(6). TFP(5) and TFP(6) were defined in an analogous fashion using inflation adjusted user costs but land was dropped as an input for TFP(5) and both land and inventories were dropped for TFP(6).

[15] It should be noted that the resulting inflation adjusted user costs were negative for inventories in 1992 and negative for land for the years 1971-72, 1974-77, 1979-80 and 1989-1992. This means that for these years, these capital inputs were actually net outputs.

Figure 3: Alternative Inflation Adjusted Declining Balance Aggregate Capital Services Prices

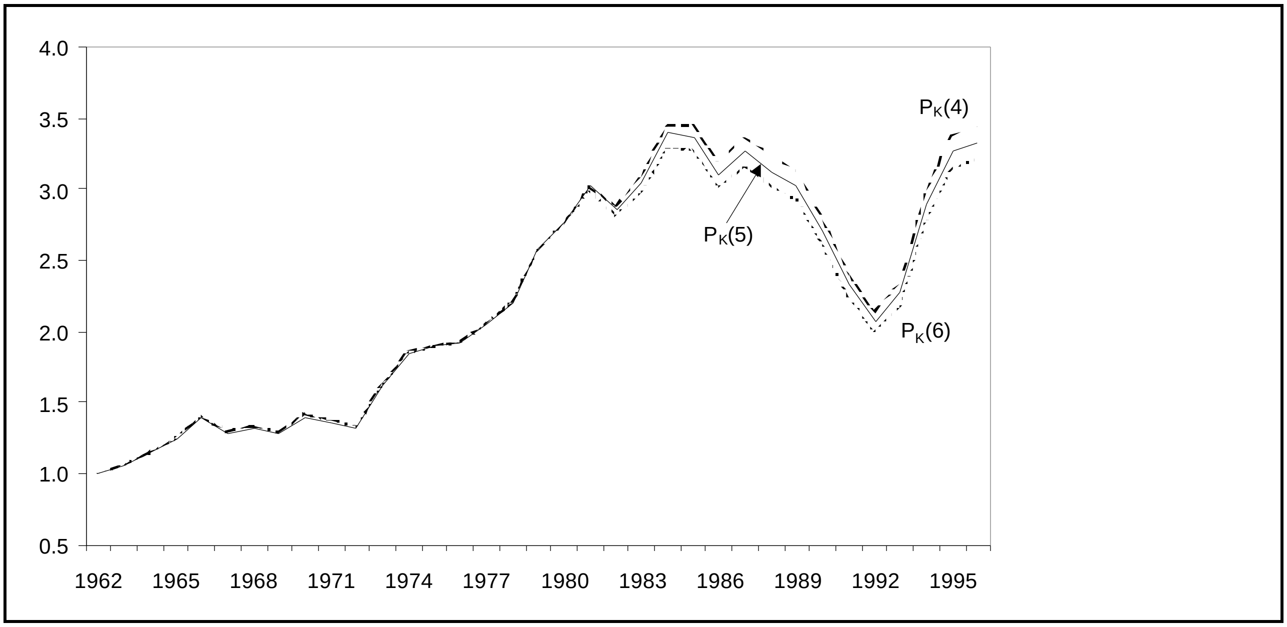
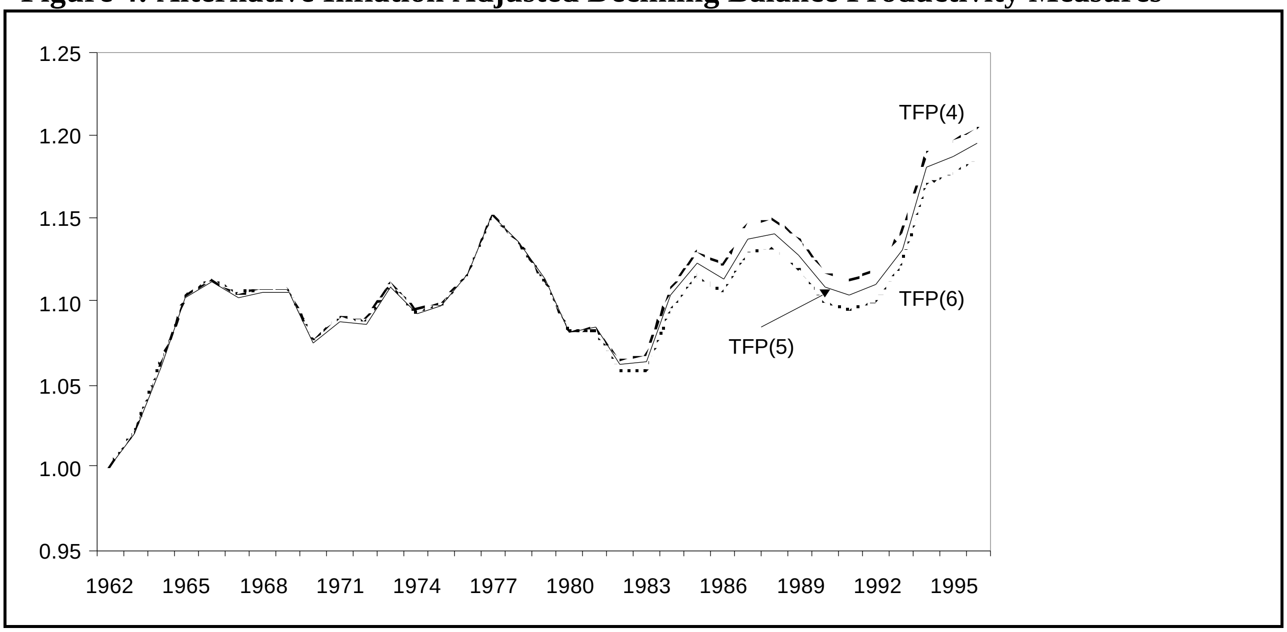


Figure 4: Alternative Inflation Adjusted Declining Balance Productivity Measures



Referring back to the g(4) column in Table 3 above, it can be seen the inflation adjusted declining balance average rate of growth for real capital services was 4.42% per year which is considerably higher than the corresponding average growth rate for real capital services for Model 1, which was 3.89% per year. What accounts for this major difference? From the Data Appendix, it can be verified that the price of land increased the most rapidly of any of the price series tabled there:

the final land price was about 25 times the 1962 level.[16] Hence, the inflation adjusted user cost for land is generally much *lower* than its unadjusted counterpart, so land (which does not grow) gets a much *smaller* price weighting in the inflation adjusted capital services aggregate, leading to a *faster* growing capital services aggregate. Thus the inflation adjusted declining balance Model 4 has a *faster* growing aggregate input than the unadjusted Model 1 and hence a *lower* average rate of productivity growth (.57% per year for Model 4 compared with .68% per year for Model 1). Since adjusting for inflation reduced the importance of land in Model 4, dropping land (Model 5) made little difference in the average TFP growth rate; it decreased from .57% per year to .54% per year over the entire sample period. The further omission of inventories (Model 6) decreased the average TFP growth rate to .52% per year. For the “lost” years, 1974-1991, dropping land and inventories from the inflation adjusted declining balance depreciation Model 4 had more of an effect: the average TFP growth rate decreased from the barely positive rate of .03% per year to the negative average rate of −.06% per year, a decline of about .1 percentage points per year. For the recent “good” years, 1992-1996, all 6 declining balance models generated an average TFP growth rate of about 1.6% per year.

We turn now to our straight line depreciation models.

6. Alternative Productivity Measures for Canada Using Straight Line Depreciation

Refer back to section 6 above for information on how the vintage capital stocks I_s^t for each year t and each vintage s were constructed for each of the two reproducible capital stocks was constructed. Using equation (22) or (23) in section 5, the *straight line depreciation model year t user cost* for a reproducible capital stock component s years old can be defined as

$$(41)\ U_s^t \equiv [1 - s/N]\, P_0^t - (1+r^t)^{-1} [1 - (s+1)/N]\, P_0^t$$
$$= (1+r^t)^{-1} [r + N^{-1} - sN^{-1}r\ ]\, P_0^t$$

where N is the assumed length of life for a unit of the new asset (12 years for machinery and equipment and 29 years for nonresidential structures) and P_0^t is the year t price of a new asset. For the nonreproducible assets, we used the same user costs in Models 7 to 9 as we used in Models 1 to 3 in the previous section.

For Model 7, for each year t, we determined the real interest rate r^t by setting the operating surplus equal to the sum of the products of each vintage stock component times its user cost. This leads to a linear equation in r^t of the following form for each year t:

$$(42)\ (1+r^t)OS = \sum_{s=0}^{28} (r^t + 29^{-1} - s29^{-1}r^t)\, P_{NSs}^t\, I_{NSs}^t$$

[16] Other price growth factors were: 1.8 for machinery and equipment; 4.0 for inventory stocks; 5.2 for nonresidential structures; 5.5 for aggregate output and 8.0 for labour.

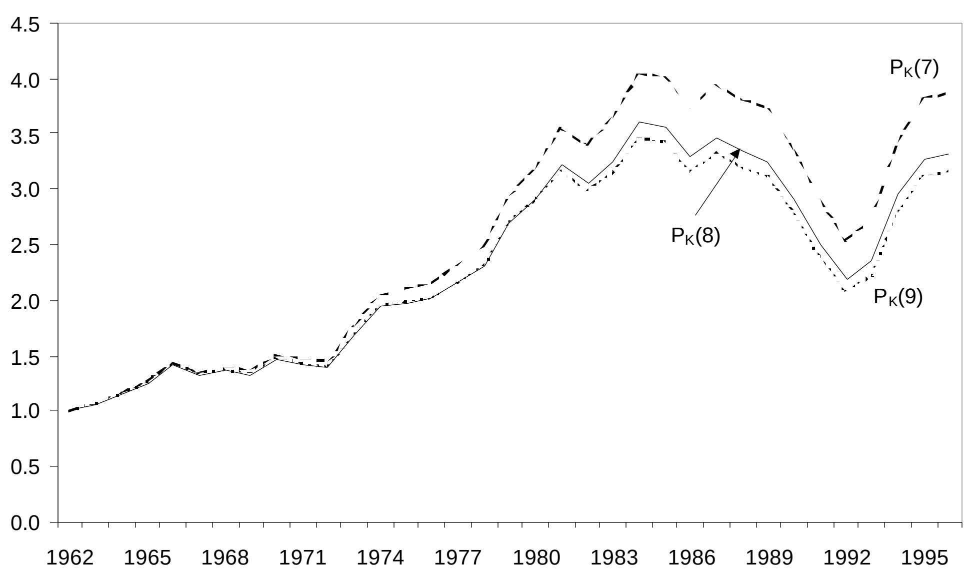
$$+ \sum_{s=0}^{11} (r^t + 12^{-1} - s12^{-1} r^t)\, P_{MEs}{}^t\, I_{MEs}{}^t + r^t P_{BAL}{}^t K_{BAL}{}^t + r^t P_{IS}{}^t K_{IS}{}^t .$$

Once the interest rate r^t has been determined for each year t, then the straight line depreciation user costs for each of the four assets can be calculated, which are of the form (45) for the two reproducible vintage capital stock components and of the form (30) for land and inventories. Then these vintage user costs can be combined with the corresponding vintage capital stock components, I_{NSs}, I_{Mes}, K_{BAL} and K_{IS}, using chain Fisher ideal indexes to form *straight line depreciation capital price and quantity aggregates*, say $P_K(7)$ and K(7). The resulting aggregate price of capital services $P_K(7)$ is graphed in Figure 5 below. We also combined the 43 vintage rental prices and quantities of capital with the price and quantity of labour, P_L and Q_L, to form the primary input aggregate, $Q_X(7)$, (using a chain Fisher ideal quantity index as usual). Once this aggregate input quantity index $Q_X(7)$ was determined, we used our aggregate output index Q_Y along with the input index in order to define the total factor productivity index, TFP(7):

(43) $TFP(7) \equiv Q_Y/Q_X(7)$.

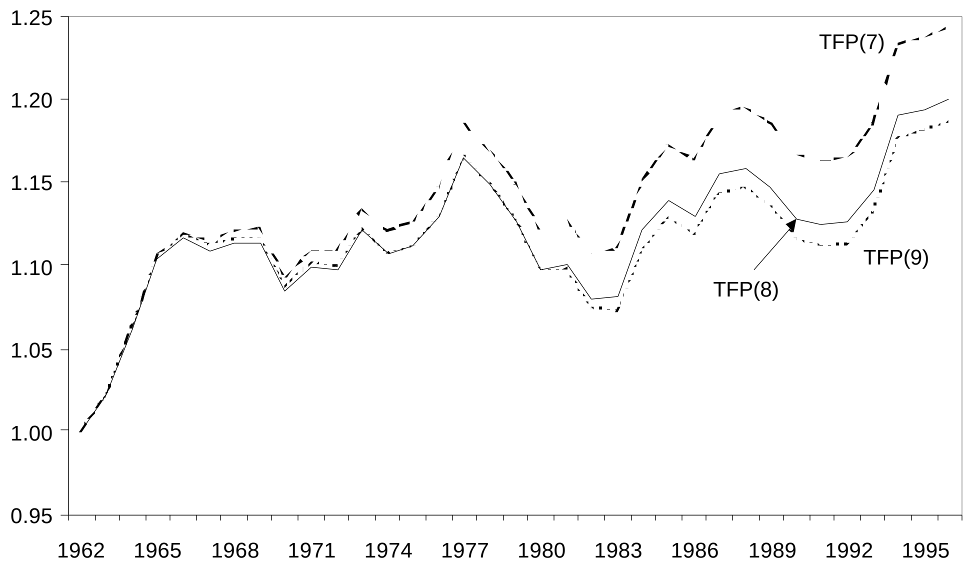
TFP(7) is graphed in Figure 6 below.

Figure 5 Alternative Straight Line Depreciation Aggregate Capital Services Prices



Models 8 and 9 are entirely analogous to Model 7 except that we dropped land from the list of inputs in Model 8 and we dropped land and inventories from Model 9.

Figure 6: Alternative Straight Line Depreciation Productivity Measures



The TFP growth factors for the years 1963-1996 for each of the three straight line depreciation models that we have considered thus far in this section are listed in the final table of the Data Appendix. However, the arithmetic averages of the three TFP growth rates for the 34 years 1963-1996, $g^t(7)$, $g^t(8)$, and $g^t(9)$, are listed in row 1 of Table 4 below.

Table 4. Averages of TFP Growth Rates for Straight Line Models (%)

	g(7)	*g(8)*	*g(9)*	*g(10)*	*g(11)*	*g(12)*
1963-96	0.66	0.55	0.52	0.55	0.52	0.50
1963-73	1.16	1.06	1.07	1.07	1.05	1.06
1974-91	0.16	0.03	-0.04	0.01	-0.02	-0.08
1992-96	1.35	1.32	1.33	1.32	1.31	1.32
average r or R	5.94	7.84	10.14	11.60	12.20	14.58
growth of K	4.06	4.54	4.69	4.59	4.69	4.80

When the straight line results in Table 4 are compared with the corresponding declining balance results listed in Table 3, we see that the results are fairly comparable for the major subperiods. In both sets of models, dropping land and then dropping inventories tends to *increase* the average growth rate of capital services and hence *decrease* the average rate of TFP growth.. However, the capital service aggregates in the straight line depreciation models tend to grow about .15% to .2% *faster* than the corresponding declining balance models. This leads to somewhat *lower* rates of TFP growth in the straight line models. This effect is particularly pronounced for the "good" years 1992-96: the average TFP growth rate *falls* from about 1.6% per year for the declining balance models to about 1.3 to 1.35% per year for the straight line models. The average real interest rate for the straight line models *increases* from 5.94% to 7.84% when land is dropped and to 10.14% when land and inventories are dropped.

We turn now to Models 10, 11 and 12, which are counterparts to Models 7,8 and 9 except we now allow for differential rates of asset inflation (as we did with Models 4,5 and 6 in the previous section). For the reproducible components of the capital stock, we switch to the inflation adjusted vintage user costs defined by (38) in the previous section. In the present context where we assume straight line depreciation, this means that the old straight line vintage user cost U_s^t defined earlier by (45) is replaced by the following *straight line depreciation inflation adjusted vintage user cost*:

(41) $U_s^t \equiv [1 - s/N]\, P_0^t - (1+R^t)^{-1}\, [1 - (s+1)/N]\, P_0^t\, (1+i^t)$

where R^t is now the year t nominal interest rate and i^t is the year t asset inflation rate for a new unit of the asset.

For Model 10, for each year t, we determined the nominal interest rate R^t by setting the operating surplus equal to the sum of the products of each vintage stock component times its inflation adjusted user cost of the form (48). This led to a linear equation in R^t similar to (46). Once the interest rate R^t has been determined for each year t, then the inflation adjusted straight line depreciation user costs can be calculated, which are of the form (48) for the two reproducible vintage capital stock components and of the form (42) (with $\delta = 0$) for land and inventories. Then these vintage user costs can be combined with the corresponding vintage capital stock components, I_{NSs}, I_{Mes}, K_{BAL} and K_{IS}, using chain Fisher ideal indexes to form *straight line depreciation inflation adjusted capital price and quantity aggregates*, say $P_K(10)$ and K(10). The resulting aggregate price of capital services $P_K(10)$ is graphed in Figure 7 below. We also combined the 43 inflation adjusted vintage rental prices and quantities of capital with the price and quantity of labour, P_L and Q_L, to form the primary input aggregate, $Q_X(10)$, (using a chain Fisher ideal quantity index as usual). Once this aggregate input quantity index $Q_X(10)$ was determined, we used our aggregate output index Q_Y along with the input index in order to define the total factor productivity index, TFP(10):

(42) $TFP(10) \equiv Q_Y/Q_X(10)$.

TFP(10) is graphed in Figure 8 below.

Figure 7: Alternative Inflation Adjusted Straight Line Depreciation Aggregate Capital Services Prices

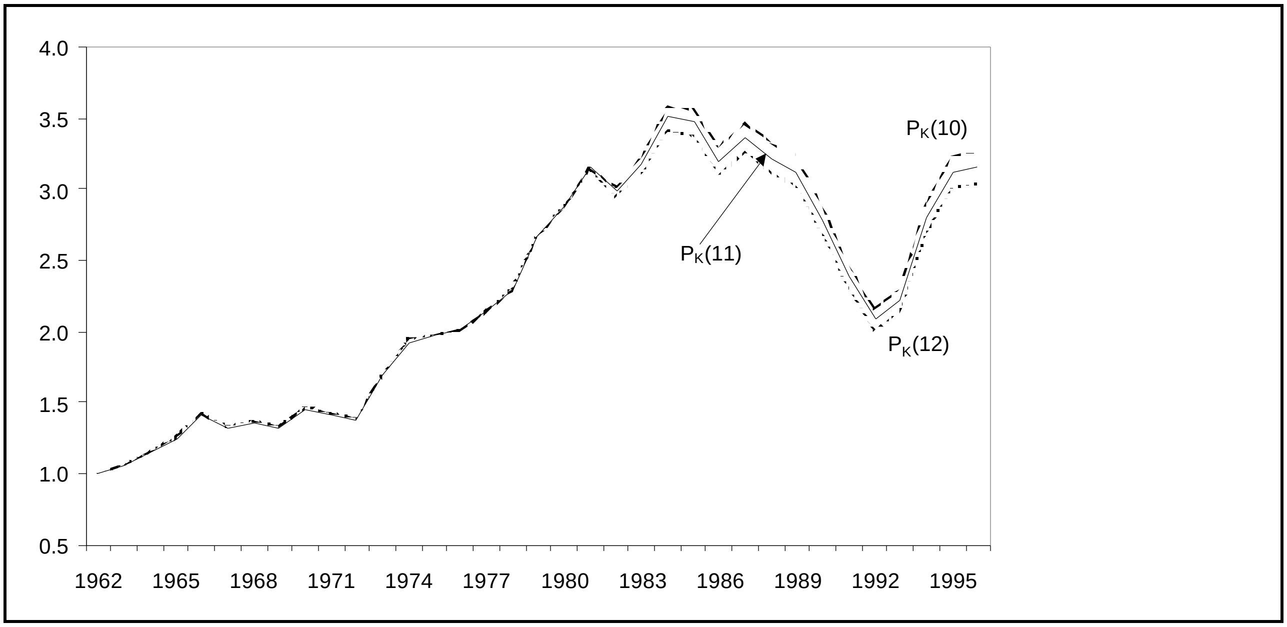
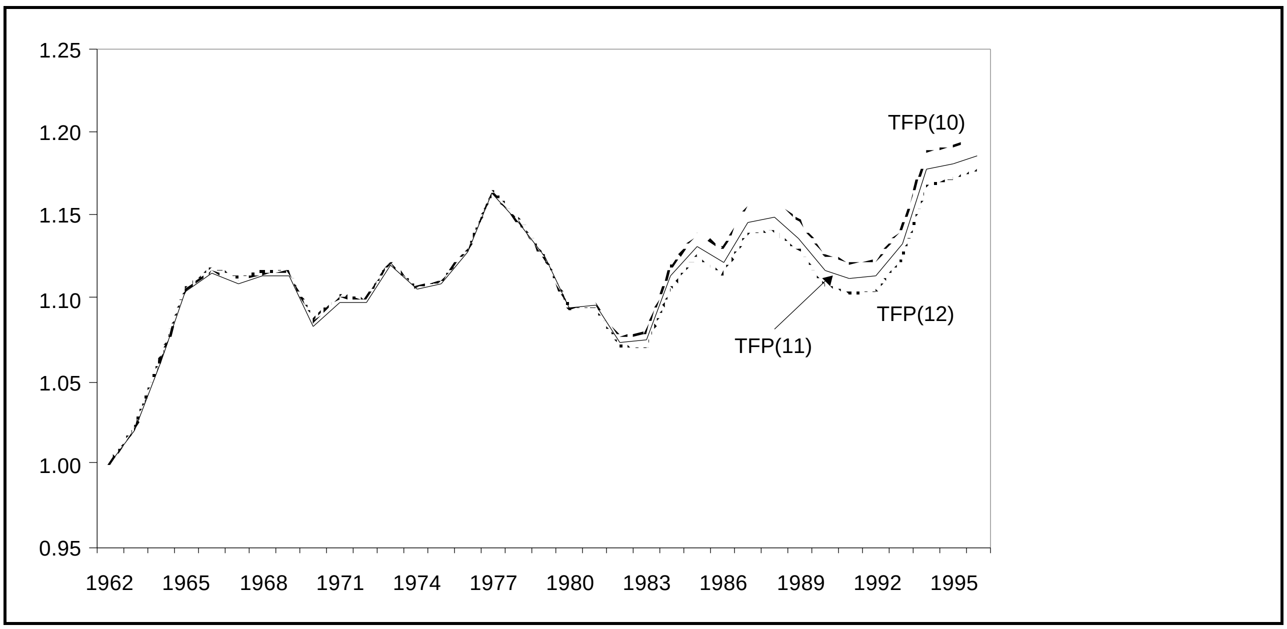


Figure 8: Alternative Inflation Adjusted Straight Line Depreciation Productivity Measures



Models 11 and 12 are entirely analogous to Model 10 except that we dropped land from the list of inputs in Model 11 and we dropped land and inventories from Model 12.

The arithmetic averages of the three straight line depreciation inflation adjusted TFP growth rates for the 34 years 1963-1996, $g^t(10)$, $g^t(11)$, and $g^t(12)$, are listed in row 1 of Table 4 above, along with the average results for the major subperiods. As was the case with the declining balance models described in the previous section, adjusting for inflation tends to *reduce* average TFP growth rates. Thus the average TFP growth rate for the entire period (with all inputs included) *falls* from .66% per year (Model 7) to .55% per year when we adjusted our straight line vintage user costs for inflation (Model 10).

We turn now to our gross capital stock models.

6. Alternative Productivity Measures for Canada Using One Hoss Shay Depreciation

Refer back to section 6 above for information on how the vintage capital stocks I_s^t for each year t and each vintage s were constructed for each of the two reproducible capital stocks was constructed. We now use formula (16) in section 4 to construct the *one hoss shay depreciation model year t user cost* for a reproducible capital stock component. For the nonreproducible assets, we used the same user costs in Models 13 to 15 as we used in Models 1 to 3 in section 7.

For Model 13, for each year t, we determined the real interest rate r^t by setting the operating surplus equal to the sum of the products of each vintage stock component times its user cost. This leads to a nonlinear equation in r^t of the following form for each year t:

$$(41)\ (1+r^t)OS = r^t\,[1-(1+r^t)^{-29}]^{-1}\,P_{NS}{}^t\,K_{NS}{}^t + r^t\,[1-(1+r^t)^{-12}]^{-1}\,P_{ME}{}^t\,K_{ME}{}^t + r^t\,P_{BAL}{}^t\,K_{BAL}{}^t + r^t\,P_{IS}{}^t\,K_{IS}{}^t$$

where $K_{NS}{}^t$ and $K_{ME}{}^t$ are the year t gross capital stocks tabled in section 6 above. The SOLVE option in SHAZAM was used to solve equation (50) for the real interest rate r^t. Once the interest rate r^t has been determined for each year t, then the one hoss shay depreciation user costs for each of the four assets can be calculated, which are of the form (16) for the two reproducible vintage capital stock components and of the form (30) for land and inventories. Then these user costs can be combined with the corresponding capital stock components, K_{NS}, K_{ME}, K_{BAL} and K_{IS}, using chain Fisher ideal indexes to form *one hoss shay depreciation capital price and quantity aggregates*, say $P_K(13)$ and K(13). The resulting aggregate price of capital services $P_K(13)$ is graphed in Figure 9 below. We also combined the one hoss shay rental prices and quantities of capital with the price and quantity of labour, P_L and Q_L, to form the primary input aggregate, $Q_X(13)$, (using a chain Fisher ideal quantity index as usual). Once this aggregate input quantity index $Q_X(13)$ was determined, we used our aggregate output index Q_Y along with the input index in order to define the total factor productivity index, TFP(13):

Figure 9: Alternative One Hoss Shay Depreciation Aggregate Capital Services Prices

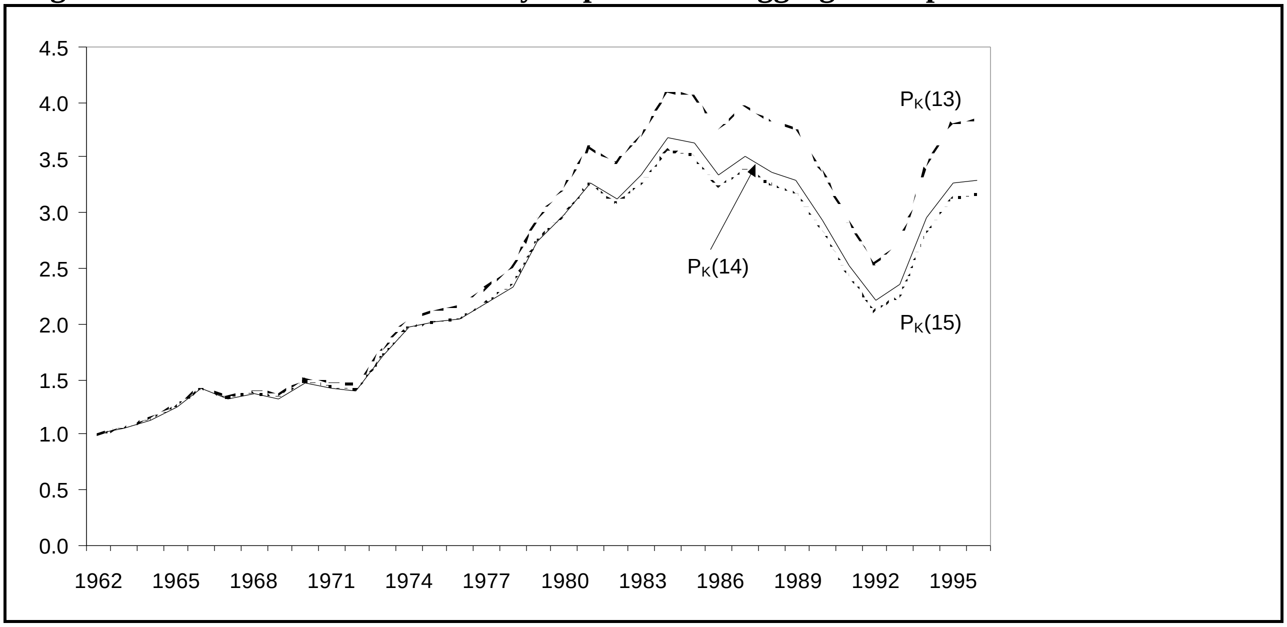
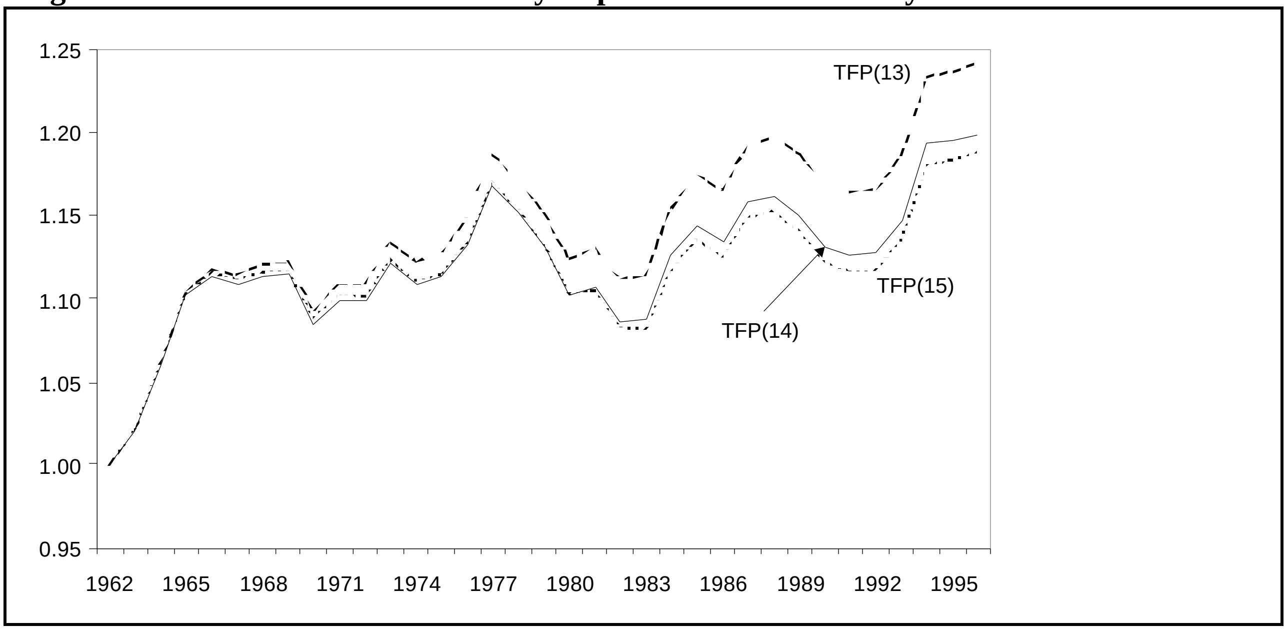


Figure 10: Alternative One Hoss Shay Depreciation Productivity Measures



(42) $TFP(13) \equiv Q_Y/Q_X(13)$.

TFP(13) is graphed in Figure 10.

Models 14 and 15 are entirely analogous to Model 13 except that we dropped land from the list of inputs in Model 14 and we dropped land and inventories from Model 15.

The TFP growth factors for the years 1963-1996 for each of the three one hoss shay depreciation models that we have considered thus far in this section are listed in the final table of the Data Appendix. However, the arithmetic averages of the three TFP growth rates for the 34 years 1963-1996, $g^t(13)$, $g^t(14)$, and $g^t(15)$, are listed in row 1 of Table 5 below.

Table 5: Averages of TFP Growth Rates for One Hoss Shay and Other Models (%)

	g(13)	*g(14)*	*g(15)*	*g(16)*	*g(17)*	*g(18)*
1963-96	0.65	0.55	0.52	0.59	0.57	0.96
1963-73	1.16	1.07	1.08	0.99	1.09	1.03
1974-91	0.16	0.04	-0.02	0.06	0.05	0.71
1992-96	1.31	1.26	1.26	1.64	1.33	1.73
average r or R	5.72	7.23	8.76	—	—	—
growth of K	4.08	4.55	4.68	4.29	4.43	2.55

When the gross capital stock results in in the first 3 columns of Table 5 are compared with the corresponding straight line results listed in the first 3 columns of Table 4, we see that the results are surprisingly close for the major subperiods. In both sets of models, dropping land and then dropping inventories tends to *increase* the average growth rate of capital services and hence *decrease* the average rate of TFP growth. The only major difference between the first 3 columns of Tables 4 and the corresponding columns in Table 5 are in the average real interest rates: they tended to be *lower* in the gross capital stock models.

Differential rates of asset inflation can be introduced into the one hoss shay model of depreciation. In the no inflation model of section 4 above, the key equation was (15), which gave the relationship between the price of a new asset, P_0, and its user cost, U_0. With a constant rate of inflation expected in future periods, so that the ratio of next period's new asset price to this period's price is expected to be (1+i), and with a constant nominal interest rate R, the new relationship between P_0 and U_0 is:

(41) $P_0 = U_0 + (1+R)^{-1}(1+i)U_0 + (1+R)^{-2}(1+i)^2U_0 +\ldots+ (1+R)^{-N+1}(1+i)^{N-1}U_0$

where N is the length of life of a new asset. Equation (52) says that the price of a new asset should be equal to the discounted stream of future expected rentals. Using (52) to solve for U_0 in terms of P_0 leads to the following *inflation adjusted one hoss shay user cost,* which replaces formula (16):

(42) $U_0 = P_0[(1+R)(1+i)^{-1} - 1](1+i)(1+R)^{-1}[1 - (1+R)^{-N}(1+i)^N]^{-1}$.

It is now possible to repeat Models 13-15, using the inflation adjusted user costs defined by (53) for the reproducible capital stock components in place of the earlier user cost formula (16).

However, given the nonlinearity of (53), we did not follow this path. If the one hoss shay model of depreciation were true, then annual rental and leasing rates for reproducible assets would be *constant* across vintages at any given point in time. Thus an old asset would rent for the same price as a new asset. This does not seem to be consistent with the "facts" and thus we do not believe it is worth spending a lot of time on one hoss shay models.

We conclude this empirical part of our paper by computing two additional capital services aggregates. For our first additional capital aggregate, we took our declining balance estimates for the two reproducible capital stock components tabled in section 6 above, K_{NS} and K_{ME}, and formed a chained Fisher ideal aggregate of these two stocks, using the investment prices P_{NS} and P_{ME} as price weights in the index number formula. We then divided the resulting stock aggregate, K(16) say, into the operating surplus OS to obtain a corresponding implicit price, $P_K(16)$ say. $P_K(16)$ is graphed in Figure 11 below, along with our first declining balance aggregate capital services price $P_K(1)$ for comparison purposes. We then combined this capital aggregate with the price and quantity of labour, P_L and Q_L, in another chained Fisher ideal aggregation in order to form an input aggregate, $Q_X(16)$. Note that land and inventory stocks are omitted from this input aggregate. Once this aggregate input quantity index $Q_X(16)$ was determined, we used our aggregate output index Q_Y along with this input index in order to define the total factor productivity index, TFP(16):

(43) $TFP(16) \equiv Q_Y/Q_X(16)$.

TFP(16) is graphed in Figure 12 below along with our first declining balance total factor productivities, TFP(1), for comparison purposes.

For our second additional capital aggregate, we took our gross capital stock estimates for the two reproducible capital stock components tabled in section 6 above and formed a chained Fisher ideal aggregate of these two stocks, using the investment prices P_{NS} and P_{ME} as price weights in the index number formula. We then divided the resulting stock aggregate, K(17) say, into the operating surplus OS to obtain a corresponding implicit price, $P_K(17)$ say. $P_K(17)$ is graphed in Figure 11 below. We then combined this capital aggregate with the price and quantity of labour, P_L and Q_L, in another chained Fisher ideal aggregation in order to form an input aggregate, $Q_X(17)$. Note that land and inventory stocks are omitted from this input aggregate. Once this aggregate input quantity index $Q_X(17)$ was determined, we used our aggregate output index Q_Y along with this input index in order to define the total factor productivity index, TFP(17):

(44) $TFP(17) \equiv Q_Y/Q_X(17)$.

Figure 11: Some Capital Services Price Aggregates

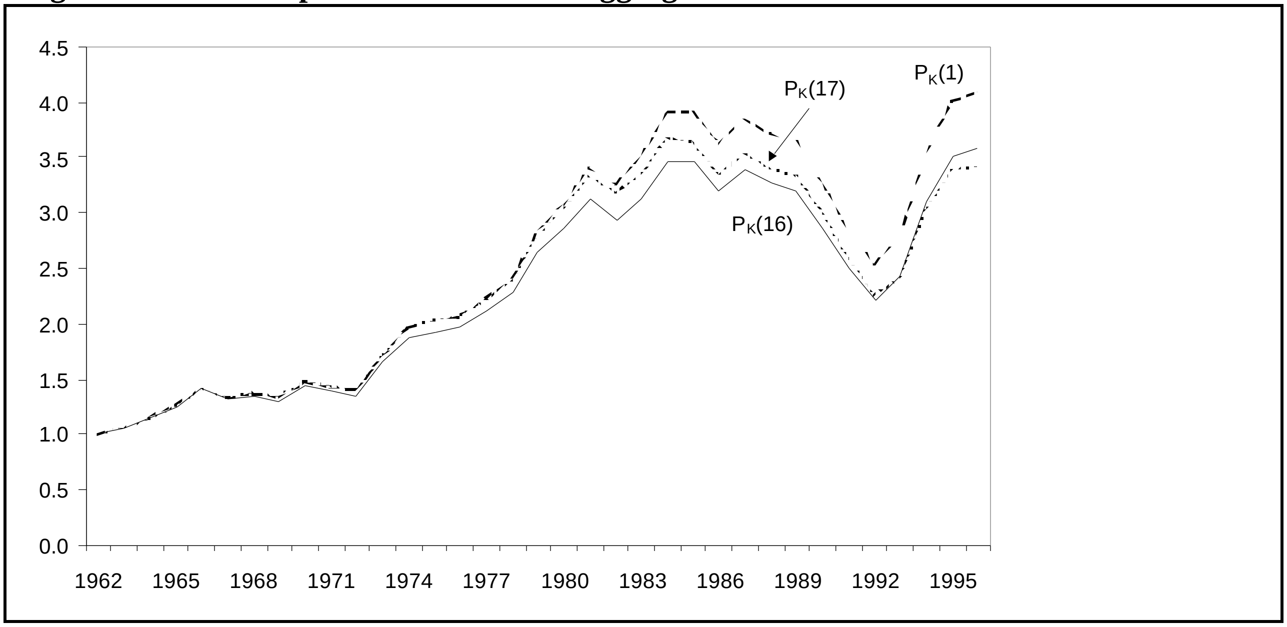
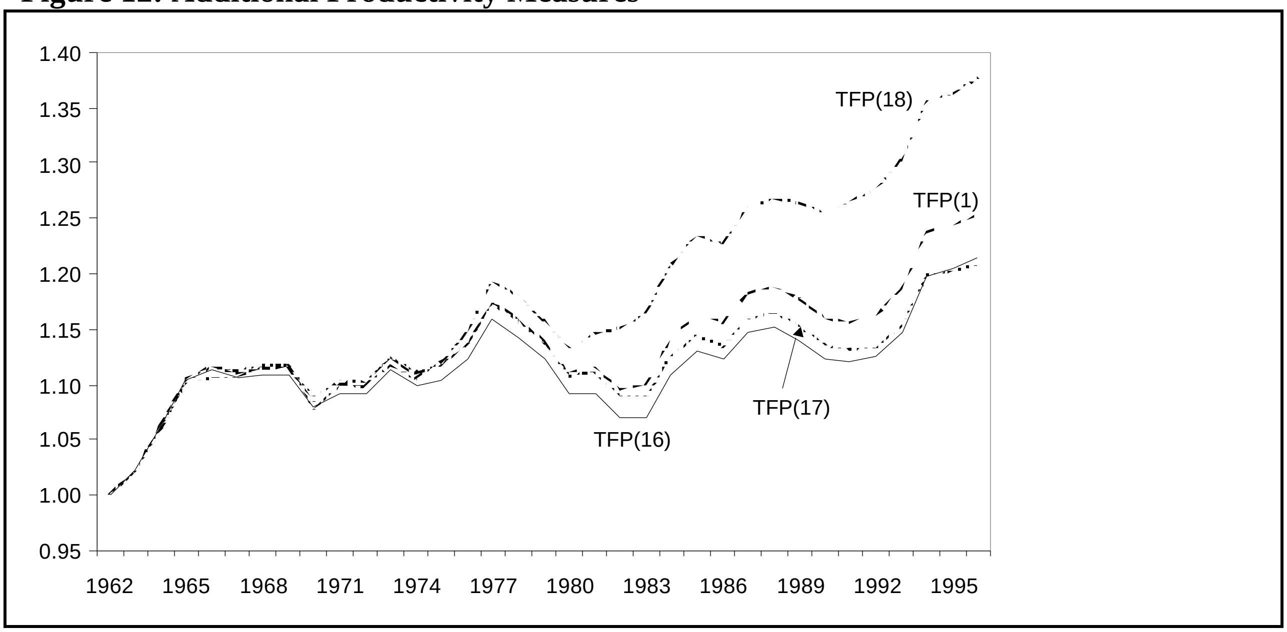


Figure 12: Additional Productivity Measures



TFP(17) is graphed in Figure 12. It can be seen that these last two TFP concepts (with 'incorrect' weighting) lead to a somewhat slower rate of TFP improvement over the entire sample compared to the no inflation declining balance concept, TFP(1).

Our final miscellaneous productivity measure is *labour productivity* TFP(18) defined as our output aggregate Q_Y divided by our measure of labour input Q_L.[17] It is graphed in Figure 12.

[17] This measure was normalized to equal 1 in 1962.

The final column in Table 5 shows that the average labour productivity growth rate over the 34 years in our sample was .96% per year which is almost twice as big as our typical average TFP growth rate. However, by international standards, this is a rather low rate of growth for labour productivity.

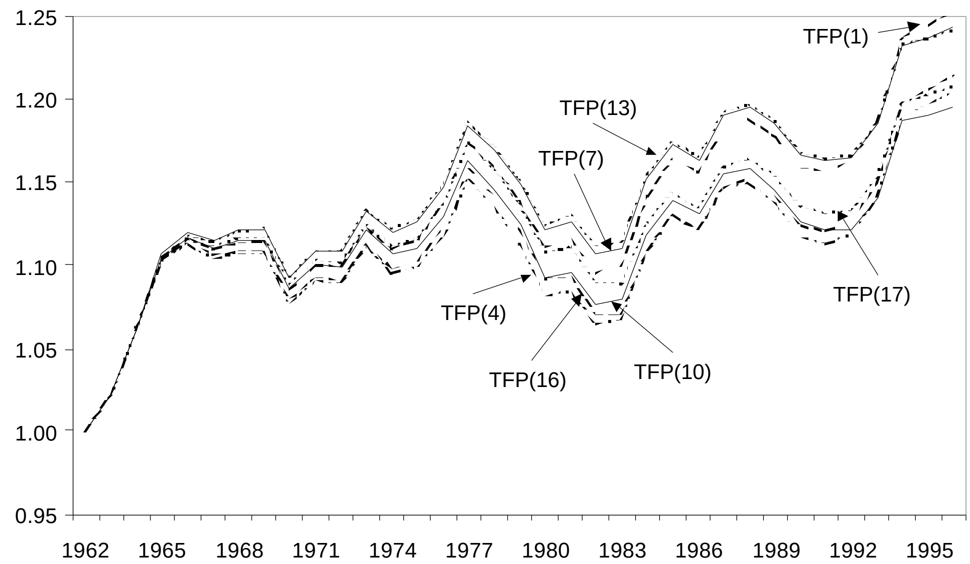
The average rates of TFP growth for our "incorrectly" weighted declining balance productivity measure TFP(16) and our "incorrectly" weighted gross capital stock productivity measure TFP(17) for the entire sample period was .59% per year and .57% per year respectively; see columns 4 and 5 in Table 5 above. These average growth rates are between those for the no inflation declining balance Models 1 and 3 (.68% and .55%) and the no inflation gross stock Models 13 and 15 (.65% and .52%). Thus our incorrectly weighted models led to productivity estimates that were fairly close to the estimates from the "correctly" weighted models.

10. Conclusion

We have shown that neglecting land and inventories leads to a decline in average TFP growth rates in Canada of about .1% per year, which is not large in absolute terms, but *is* large in relative terms, since the average growth rate for total factor productivity in Canada only averaged .5 to .6% per year over the years 1963-96. However, once land and inventories are included in the capital aggregate, the differences in average TFP growth rates between the various depreciation models (declining balance, one hoss shay and straight line) proved to be surprisingly small, whether we allowed for differential rates of asset inflation or not.

We summarize our results in Figure 13, where we graph our productivity estimates for Model 1 (declining balance depreciation including all assets), Model 4 (declining balance depreciation including all assets with inflation adjustments), Model 7 (straight line depreciation including all assets), Model 10 (straight line depreciation including all assets with inflation adjustments), Model 13 (one hoss shay depreciation including all assets), Model 16 (declining balance depreciation but with "incorrect" stock weights instead of user cost weights and excluding land and inventories) and Model 17 (one hoss shay depreciation but with "incorrect" stock weights instead of user cost weights and excluding land and inventories). It can be seen that the three 'correctly' weighted models with no inflation were relatively close to each other and finished up about 5 percentage points higher than the two 'correctly' weighted models with inflation adjustments, TFP(4) and TFP(10), and the two 'incorrectly' weighted measures, TFP(16) and TFP(17). We note the highest estimate of TFP in 1996 is given by Model 1 (a 25.29% increase from 1962) and the lowest estimate is given by Model 10 (a 19.63% increase). This is not a huge range of variation.

Figure 13: Alternative Total Factor Productivity Estimates for Canada



All of our productivity estimates paint more or less the same dismal picture of Canada's productivity performance. During the pre OPEC years, 1962-73, TFP growth proceeded at the satisfactory rate of about 1 per cent per year. Then for the 18 "lost" years, 1974-1991, TFP growth was close to 0 on average. Fortunately, there appears to have been a strong TFP recovery in recent years; TFP growth averaged somewhere between 1.6% and 1.3% per year for the 5 years 1992-96.[18]

There are many problems associated with the measurement of capital that were not discussed in this paper. Some of these problems are:

- We have discussed only ex post user costs, which we think is appropriate when measuring the productivity performance of a firm or industry or country. However, for many other purposes (such as econometric modeling), ex ante or expected user costs are more relevant.
- The user costs that were defined neglected the complications due to the business income tax and other taxes on capital. Essentially, our user costs assume that these taxes just reduce the pretax real or nominal rate of return.[19]
- We have not related depreciation to the utilisation of the asset.
- We have discussed only the easy to measure components of the capital stock. Other components that were not discussed include resource stocks, knowledge stocks and infrastructure stocks.

---

[18] Diewert and Fox (1999) hypothesized that the world wide TFP slowdown that occurred in OECD countries around 1973 was probably related to the big increase in inflation that occurred around that time. Inflation was high in Canada during the years 1974-1991 and then low in recent years. Thus Canada's recent productivity recovery is consistent with the Diewert and Fox hypothesis.

[19] Thus our r^t or R^t are returns that include these business taxes.

- We have not modeled the role of research and development expenditures and of knowledge spillovers.
- We have not discussed the problems involved in measuring capital when there are quality improvements in new units of the capital stock.

However, we hope that our presentation of alternative models of depreciation will be helpful to business and academic economists who find it necessary to construct capital aggregates in the course of their research. We also hope that our exposition will be helpful to statistical agencies who may be contemplating adding a productivity module to their economic statistics. We have shown that it is relatively easy to do this once accurate information on asset lives (or declining balance depreciation rates) are available.

Data Appendix

In this appendix, we will briefly describe our sources and list the data actually used in our capital stock and productivity computations.

We begin by describing the construction of our aggregate output variable. From Tables 52 and 53 of the Statistics Canada publication, *National Income and Expenditure Accounts, Annual Estimates 1984-1995* (and other years), we were able to construct consistent estimates for 19 categories of consumer expenditures for the years 1962-1996. The 19 categories were: (1) food and nonalcoholic beverages; (2) alcoholic beverages; (3) tobacco products; (4) clothing, footwear and accessories; (5) electricity, natural gas and other fuels; (6) furniture, carpets and household appliances; (7) semidurable household furnishings plus reading and entertainment supplies; (8) nondurable household supplies, drugs and sundries, toilet articles and cosmetics; (9) medical care, hospital care and other medical care expenses; (10) new and (net) used motor vehicles plus motor repairs and parts; (11) motor fuels and lubricants; (12) other auto related services plus purchased transportation; (13) communications; (14) recreation equipment, jewelry, watches and repairs; (15) recreational services; (16) educational and cultural services; (17) financial, legal and other services; (18) expenditures on restaurants and hotels and (19) other services (laundry and dry cleaning, domestic and child care services, other household services and personal care). Note that we do not include consumption of housing services in the above list of consumer goods and services. We will also exclude the stock of dwellings from our list of market sector capital inputs. The price series for the above 19 components of consumer expenditure contain various commodity taxes, which are revenues for government but are not revenues for private producers. Thus we attempted to remove these commodity taxes from the above price series using information contained in the Statistics Canada publication, *The Input-Output Structure of the Canadian Economy 1961-1981* and other years. Additional information from the Statistics Canada publication, *The Canadian Economic Observer* for May 1989 and other Statistics Canada sources was used in order to construct final estimates of commodity taxes for the above 19 final demand consumption categories. We note that we were unable to allocate all indirect taxes and subsidies to the appropriate categories so our market sector output aggregate will be subject to some measurement error.

We turn now to a description of our international trade data. It should be mentioned that our treatment of international trade follows that of Kohli (1978)(1991). In this treatment, exports are produced by the market sector and all imports flow into the market sector as intermediate inputs. These import inputs are either physically transformed by domestic producers or they have domestic value added to them through domestic transportation, storage or retailing activities. When we construct our market sector output aggregate, we index import quantities with a negative sign in keeping with national income accounting conventions.

In forming consistent series for disaggregated export and import components, the principal data source was the Statistics Canada CANSIM matrices 6566 and 6541. These matrices provide current and constant price series for 11 export and 11 import components for the years 1971 to 1993.

For exports, the 11 components were aggregated into the following 5 categories on the basis of similarities in price movements (Hicks aggregation): (1) Forestry; (2) Energy; (3) Equipment; (4) Other goods; and (5) Services. The Forestry and Energy categories were formed directly from the equivalent CANSIM series. The Equipment category is an aggregate of the CANSIM series for Machinery and equipment and Automotive products. The residual Other goods exports category is an aggregate of the CANSIM Agricultural and fish products, Industrial goods and materials and Other consumer goods components. The Services category is an aggregate of the CANSIM Travel services, Transportation services and Commercial services components plus the value of Government services obtained from Statistics Canada's *Canada's Balance of International Payments, 1926 to 1996* and first quarter 1997, Catalogue No. 67-001-XPB, Table 13. The Government services variable mainly comprises expenditures by foreign governments in Canada. It is excluded from the CANSIM series but we assume these purchases are from the Canadian business sector and so should be included in our export series. The eleventh CANSIM component – Financial Intermediation services – was omitted, as it does not accurately represent the movement of goods or services. Rather, it is largely a financial balancing item. Export price indexes were formed using chained Fisher indexes of the component implicit prices from the CANSIM current and constant price matrices. As our complete database runs over the years 1962 to 1996, we had to backdate our 5 export categories to 1962 using a variety of sources. In order to extend the CANSIM series from 1993 to 1996, again a variety of sources were used. The values and prices of exports of the four goods components were updated from 1993 to 1996 using Statistics Canada's *Canadian Economic Observer*, Catalogue No. 11-010-XPB, Tables 18 and 19. The July 1996 and November 1997 issues spanned the four year period. Values and current weighted price indexes were presented for exports of Agriculture and fish, Energy, Forestry, Industrial goods and materials, Machines and equipment, Automotive and Consumer goods. The value of the four services items making up the Services export component were updated from Statistics Canada's *Canada's Balance of International Payments,* 1926 to 1996 and first quarter 1997, Catalogue No. 67-001-XPB, Table 13. The price of the overall Services export component was updated to 1996 using the price of non-merchandise exports obtained from CANSIM matrix 6628, Current and constant dollar estimates of non-merchandise exports and imports from the National Accounts.

The values and price indexes for the five export categories described above are measured at the border and so exclude export taxes paid by producers. To derive exports in producers' prices we needed to estimate export taxes for each of our five export components. The only export components, which had significant export taxes, are Energy and Services exports. Between 1973 and 1985 Canada imposed an Oil Export Charge. Values of this tax were obtained from Statistics Canada's *National Income and Expenditure Accounts, Annual Estimates, 1926-83*, Catalogue No. 13-531, Table 51. For Services exports, we assumed that three of its components were taxed: Travel, Freight and shipping, and Government services. We assumed that Travel and Government services exports were each made up of half expenditure on fuel and half expenditure on hotels and restaurants. Consequently, the commodity tax rates for Fuel and Hotels and restaurants were each applied to half the expenditure in each of these components. We assumed that half of Freight and shipping exports were made up of fuel expenses and subject to the commodity tax rate for Fuel. Other services exports were assumed not to be taxed.

For imports, the 11 components in CANSIM matrices 6566 and 6541 were aggregated into the following 4 categories on the basis of similarities in price movements (Hicks aggregation): (1) Forestry and other; (2) Energy; (3) Equipment; and (4) Services. The Energy and Equipment categories are formed directly from the equivalent CANSIM series. The Services category is an aggregate of the Travel, Transportation and Commercial services components. The Forestry and other imports category is the aggregate of the Forestry, Agricultural and fish products, Industrial goods and materials, Automotive products and Other consumer goods components. Financial Intermediation services are again omitted. Import price indexes were formed using chained Fisher indexes of the component implicit prices from the CANSIM current and constant price matrices. As was the case with exports, our 4 import series were backdated to 1962 using a variety of sources. The values and prices of imports of the three goods components were updated from 1993 to 1996 using Statistics Canada's *Canadian Economic Observer*, Catalogue No. 11-010-XPB, Tables 18 and 19. The July 1996 and November 1997 issues spanned the four year period. Values and current weighted price indexes were presented for imports of Agriculture and fish, Energy, Forestry, Industrial goods and materials, Machines and equipment, Automotive and Consumer goods. The value of the three services items making up the Services import component were updated from Statistics *Canada's Canada's Balance of International Payments*, 1926 to 1996 and first quarter 1997, Catalogue No. 67-001-XPB, Table 13. The price of the overall Services import component was updated to 1996 using the price of non-merchandise imports obtained from CANSIM matrix 6628, Current and constant dollar estimates of non-merchandise exports and imports from the National Accounts. These import values and prices are measured at the border and so exclude import duties. To derive imports in producers' prices we needed to estimate import duties for each of our four import components. Total import duties for the period 1962-83 were obtained from Statistics Canada's *National Income and Expenditure Accounts, Annual Estimates, 1926-83*, Catalogue No. 13-531, Table 51. Import duties for 1984-95 were obtained from *National Income and Expenditure Accounts, 1984-95*, Catalogue No. 13-201-XPB, Table 44. Total import duties for 1996 were estimated by assuming the same aggregate tariff rate applied as that observed in 1995. An estimate of the duty paid on Equipment imports was obtained from various issues of the Canadian Tax Foundation's *The National Finances* for the years 1964-85. The following categories from the End products, inedible, category were allocated to Equipment imports: General purpose industrial machinery, Special industry machinery, Road motor vehicles, Communication and related equipment, Electric lighting, distribution and control equipment and Office machines and equipment. Values for 1969, 1974 and 1984 were estimated by interpolating implied tariff rates. Equipment import duties for the remaining periods, 1962-63 and 1986-96, were estimated by linking the implied Equipment tariff rate to movements in the total imports implied tariff rate. Information in *The National Finances* indicated there were no import duties on Energy imports. We assume there are no tariffs applying to Services imports so the remaining duties are allocated to the Forestry and other imports component. Between 1973 and 1985 Canada subsidized oil imports. The amount of the Oil Import Compensation Charge was obtained from *The National Finances*. This enters as a subsidy, or negative tax, on Energy imports for these years. The subsidy rate for 1982 appeared anomalous and was replaced by an interpolated rate.

The government purchases intermediate inputs and investment goods from the market sector so it is necessary to form estimates for these components of market sector output. We have data on the value and price of total government consumption and government wages payments from the *OECD Economic Outlook* database (Econdata 1997). We derive the value of government purchases of intermediates by subtracting government wages payments from total government consumption. Having derived the value and price of government purchases of intermediates, we combine this with the value and price of government investment in fixed capital from CANSIM matrices 6828 and 6836, respectively, using a chain Fisher index. Our next task was to derive producers' price series for the two components of government purchases. Indirect taxes paid on total government purchases from the market sector (both intermediates and fixed capital expenditure) were obtained from the final demand matrices of Statistics Canada's *Input–Output Tables* for the years 1962 to 1992. An indirect tax rate was formed by taking the ratio of indirect taxes paid to the value of total government purchases from the market sector. The tax rate for 1992 was assumed to also apply for the remaining four years, 1993 to 1996. Producers' price indexes were then formed by multiplying the government intermediates and investment price indexes by one minus the indirect tax rate.

The final components of our market sector output aggregate are the investment components and the change in inventories. From *The Canadian Economic Observer, Historical Statistical Supplement, 1997/98*, we obtained investment in nonresidential structures and in machinery and equipment in current and constant dollars for the years 1962-1997. The resulting price (P_{NS} and P_{ME}) and quantity series (I_{NS} and I_{ME}) are listed in Table A1 below.

The construction of price and quantity series for inventory change is not straightforward. From *The Canadian Economic Observer, Historical Statistical Supplement, 1997/98*, we obtained estimates of inventory in current and constant dollars for the years 1962-1997. The resulting price series is listed in Table A1 below as P_{INA} (price of inventories using national accounts data). It can be seen that this price series for inventory change is not credible as a measure of the average level of inventory prices in a given year. Hence, we will use the Statistics Canada *National Balance Sheet Accounts, Annual Estimates, 1996* for estimates of the total stock of inventories (in current dollars) held at the end of the year for the years 1964 to 1996. This in turn is equal to the beginning of the year stock of inventories held by the market sector (the government's holdings of inventories was negligible) for the years 1965-1997. This series can be extended back to stocks held at the beginning of 1962 using the *National Balance Sheet Accounts* for an earlier year. Constant dollar stocks of inventories are available for the end of years 1961-1982 from the *National Balance Sheet Accounts, Annual Estimates, 1984* and for the end of years 1984-1993 from the *National Balance Sheet Accounts, Annual Estimates, 1994*. The resulting beginning of the year inventory stock price and quantity series for the years 1962-1994 are listed in Table A1 as P_{IS} and K_{IS}. In order to extend our inventory price series to 1997, we linked P_{IS} to the (erratic) national accounts series P_{INA} at the year 1992, which was the base year for the constant dollar inventory change series. The final 3 entries in the P_{IS} series reflect this linking procedure. The resulting P_{IS} series looks quite reasonable. Of course, once we have our price series for beginning of the year inventory stocks, P_{IS}, it can be divided into the balance sheet value estimates to obtain the quantity series for beginning of the year inventory stocks, K_{IS}, for all 37 years in our sample, 1962-1997. Once K_{IS} has been determined, then inventory change, Q_{IC} for the 36 years 1962-1996 can be obtained by differencing the stock series, K_{IS}.

Table A1: Market Sector Output Data for Canada; 1962-1997

Year	P_{NS}	P_{ME}	P_{IS}	P_{INA}	P_Y
1962	1.0000	1.0000	1.0000	1.0000	1.0000
1963	1.0285	1.0280	0.9866	0.9918	1.0241
1964	1.0579	1.0695	1.0288	1.0909	1.0456
1965	1.1168	1.1114	1.0451	1.0445	1.0842
1966	1.1843	1.1476	1.0718	1.0325	1.1422
1967	1.2355	1.1398	1.0970	1.0960	1.1797
1968	1.2492	1.1387	1.1180	1.1506	1.2306
1969	1.3199	1.1692	1.1376	1.0938	1.2936
1970	1.3872	1.2245	1.1618	1.3038	1.3495
1971	1.4713	1.2576	1.1638	1.0937	1.4027
1972	1.5549	1.3077	1.1830	1.2551	1.4800
1973	1.7235	1.3433	1.2974	1.3728	1.6212
1974	2.0439	1.4864	1.5420	1.4454	1.8776
1975	2.2837	1.6650	1.8820	1.7817	2.1021
1976	2.4139	1.7641	1.9290	1.7078	2.2773
1977	2.5428	1.8706	1.9544	2.0536	2.4025
1978	2.7279	1.9467	2.0555	1.6277	2.5687
1979	2.9751	2.0667	2.2834	1.8909	2.8554
1980	3.3367	1.9616	2.6506	4.2658	3.2014
1981	3.7133	1.9860	3.0073	2.5485	3.5656
1982	3.9903	2.1330	3.2945	2.7584	3.8687
1983	3.9631	2.1127	3.3440	2.7320	4.0607
1984	4.1112	2.0873	3.4455	2.9947	4.1719
1985	4.2349	2.0339	3.5308	2.4212	4.2571
1986	4.2977	2.0313	3.5633	2.8847	4.3085
1987	4.4991	1.9851	3.6102	2.7174	4.4881
1988	4.7563	1.9477	3.7149	4.4340	4.6604
1989	4.9655	1.9397	3.8702	2.8741	4.8720
1990	5.1192	1.9237	3.8954	4.2322	5.0121
1991	5.0094	1.7483	3.8846	3.1432	5.0985
1992	4.9730	1.6926	3.7546	3.1024	5.1114
1993	5.0365	1.7181	3.9027	3.9837	5.1431

1994	5.2066	1.7630	4.0362	2.2776	5.2164
1995	5.2717	1.7845	4.1109	3.3968	5.3585
1996	5.2479	1.7719	4.1564	3.4343	5.4999
1997			4.0474	3.3443	

Table A1: Market Sector Output Data for Canada; 1962-1997 (cont'd)

Year	I_{NS}	I_{ME}	K_{IS}	Q_{IC}	Q_Y
1962	2560.0	2368.0	13811.3	791.8	32312.7
1963	2647.5	2570.1	14603.1	467.5	33750.7
1964	3060.7	3023.8	15070.7	461.1	36237.1
1965	3338.2	3551.4	15531.8	1145.3	38933.8
1966	3830.9	4229.7	16677.1	1176.2	41905.0
1967	3631.8	4321.8	17853.4	352.7	43435.4
1968	3601.6	4022.9	18206.1	759.0	44962.3
1969	3605.6	4404.0	18965.1	1579.5	46565.0
1970	3955.5	4489.0	20544.6	376.3	48619.9
1971	4080.2	4623.2	20920.9	316.5	50250.7
1972	4063.3	4999.5	21237.5	120.1	51961.9
1973	4384.0	6119.1	21357.5	428.7	55649.0
1974	4652.5	6893.1	21786.2	1546.0	57391.4
1975	5261.6	7285.4	23332.3	650.7	59005.1
1976	5139.9	7603.2	23982.9	1553.2	62302.4
1977	5450.6	7584.9	25536.1	2033.7	65853.2
1978	5591.9	8207.6	27569.8	1059.5	67678.3
1979	6311.6	9550.6	28629.3	1436.1	69943.6
1980	7030.7	11527.5	30065.5	-907.5	70814.5
1981	7565.8	13936.2	29157.9	450.0	73641.7
1982	6877.9	11750.8	29608.0	-2476.4	71606.8
1983	6309.2	11531.0	27131.5	-670.5	72619.0
1984	6242.5	12306.7	26461.1	1428.8	77445.1
1985	6556.9	14174.9	27889.8	790.5	81153.7
1986	6177.5	15712.8	28680.4	570.6	83345.0
1987	6416.4	18135.8	29251.0	929.3	87498.1
1988	7067.9	21512.3	30180.3	699.7	91071.2
1989	7285.1	23169.5	30880.0	1180.8	92786.7
1990	7302.0	22141.5	32060.7	-569.0	92968.8
1991	7065.7	22260.9	31491.8	-613.4	91988.5
1992	5962.9	22836.3	30878.4	-1149.9	92059.5
1993	5992.9	21725.6	29728.5	357.6	95077.8
1994	6521.2	23719.6	30086.1	1820.7	100532.2
1995	6473.1	25129.9	31906.8	2051.4	103633.5
1996	6752.6	26179.2	33958.2	1696.2	104886.0
1997			35654.4		

This completes our description of the construction of the components of our market sector output aggregate. The aggregate price of output, P_Y, and the aggregate quantity of output, Q_Y, were constructed as chained Fisher ideal indexes of the 19 consumption components, the 5 export components, the 4 import components, the 2 government components (investment purchases and purchases of goods and services), the 2 investment components and the inventory change component. P_Y and Q_Y are listed in Table A1.

We now describe the construction of the primary input components for the market sector of the Canadian economy.

We first describe our labour estimates. From *The Canadian Economic Observer, Historical Statistical Supplement, 1997/98*, we obtained estimates of the number of self employed workers (including unpaid family workers) from Table 8. For the years 1962-1974, we obtained the same information from various *Canada Year Books*. For the year 1975, we interpolated an estimate using the two sources. From Table 9 of *The Canadian Economic Observer, Historical Statistical Supplement, 1997/98*, we obtained estimates of total labour income and labour income paid out to public administration workers. Thus by subtraction, we obtained estimates for market sector employment income. From the Organisation of Economic Cooperation and Development's *Economic Outlook* database (Econdata 1997), we obtained data on the business compensation rate, P_L, which is an annual wage that reflects the full cost of employing a full time worker. We attributed 2/3 of this annual wage rate to the self employed and we assume that all of the self employed worked in the market sector rather than the government sector. This gave us a new (bigger) estimate of market sector total labour income. We divided this labour value series by the full time market wage rate P_L in order to obtain market sector labour input, Q_L. The series P_L and Q_L are listed in Table A2. Subtracting P_L times Q_L from the value of market sector outputs (less imports), P_Y times Q_Y, gave us estimates of the market sector's *operating surplus*, OS. This series is also listed in Table A2.

We turn now to the capital components of the input aggregate. We have already described how we used the *National Balance Sheet Accounts* for estimates of the total stock of inventories. The same balance sheets can be used to form estimates of the beginning of the year stocks of nonresidential structures, machinery and equipment and land used by the market sector. We will not go through all of the details of the construction of these series. These beginning of the year balance sheet estimates for constant 1962 dollar net nonresidential structures stocks, K_{NS}, machinery and equipment stocks, K_{ME}, and business and agricultural land stocks, K_{BAL}, are listed in Table A2 below along with the corresponding price of land, P_{BAL}. Note that we have assumed that the stock of business and agricultural land is constant. The price for a unit of the nonresidential structures is assumed to be the same as the corresponding investment price, P_{NS}. The price for a unit of the machinery and equipment stock is assumed to be the same as the corresponding investment price, P_{ME}.[20]

[20] We could have used the price deflators for nonresidential structures and for machinery and equipment that can be constructed using the constant dollar estimates of these stock components that may be found in the *National Balance Sheet Estimates*. However, we found that the resulting balance sheet price series for machinery and equipment differed substantially from the corresponding investment price deflator for machinery and equipment listed in Table A1, P_{ME}. We feel that the investment price and quantity data are more accurate than the balance sheet price

and quantity data with the exception of the national accounts change in inventories series. Thus we used the national accounts investment prices P_{NS} and P_{ME} to deflate the balance sheet capital stock values for nonresidential structures and machinery and equipment repectively.

Table A2: Market Sector Input Data for Canada; 1962-1997

Year	P_L	Q_L	*OS*	K_{NS}	K_{ME}	K_{BAL}	P_{BAL}	δ_{NS}	δ_{ME}
1962	1.0000	24118.2	8194.5	30006.6	17983.7	11743.1	1.0000	0.0578	0.0969
1963	1.0422	24657.5	8864.4	30830.7	18609.7	11743.1	1.0695	0.0417	0.1191
1964	1.0984	25555.6	9821.6	32192.4	18964.3	11743.1	1.1647	0.0743	0.1104
1965	1.1795	26382.0	11096.6	32860.3	19894.6	11743.1	1.3126	0.0540	0.1241
1966	1.2272	28284.5	13152.1	34424.6	20977.6	11743.1	1.4529	0.0249	0.0830
1967	1.3048	29314.1	12993.2	37398.8	23465.5	11743.1	1.6315	0.0108	0.0927
1968	1.3729	30086.2	14025.0	40628.4	25611.2	11743.1	1.7998	0.0797	0.1207
1969	1.4799	31107.9	14202.2	40991.6	26542.2	11743.1	1.9647	0.0375	0.1250
1970	1.4660	33670.2	16255.7	43058.0	27627.3	11743.1	2.1581	0.0571	0.1129
1971	1.5833	34141.3	16427.9	44555.9	28997.2	11743.1	2.3732	0.0302	0.1297
1972	1.7062	35323.9	16635.3	47292.0	29860.1	11743.1	2.7360	0.0959	0.1140
1973	1.8592	37185.2	21082.2	46820.1	31455.3	11743.1	3.2506	0.1145	0.1871
1974	2.1251	38782.7	25338.0	45844.7	31689.7	11743.1	4.0754	-0.0113	0.1368
1975	2.4558	39289.9	27545.4	51015.6	34248.6	11743.1	5.1559	-0.0038	0.0829
1976	2.7657	40545.3	29748.1	56471.3	38693.9	11743.1	6.1750	0.0562	0.1249
1977	3.0256	41185.8	33599.6	58439.8	41466.0	11743.1	7.1203	0.0781	0.1078
1978	3.1726	42806.5	38032.9	59325.3	44579.4	11743.1	8.2260	0.0792	0.1219
1979	3.3912	45118.9	46708.6	60218.0	47354.4	11743.1	9.5535	0.0900	-0.0008
1980	3.7144	46640.3	53461.6	61107.2	56943.2	11743.1	11.4351	0.0697	0.1244
1981	4.1680	47985.3	62573.4	63880.1	61387.2	11743.1	13.7683	0.0304	0.2081
1982	4.5917	46436.4	63803.4	69506.2	62549.6	11743.1	14.9309	-0.0206	0.0557
1983	4.8055	46587.1	71009.0	77816.8	70819.3	11743.1	14.7458	0.1067	0.1246
1984	5.0528	47808.0	81529.4	75826.6	73528.3	11743.1	14.5917	0.0714	0.0990
1985	5.3243	49086.8	84128.0	76652.6	78559.1	11743.1	14.7611	0.0504	0.1711
1986	5.4839	50703.4	81039.4	79349.1	79293.0	11743.1	14.3209	0.0893	0.1154
1987	5.8528	51853.3	89211.6	78437.9	85853.4	11743.1	15.1011	0.0865	0.1538
1988	6.2403	53625.4	89787.8	78070.0	90782.5	11743.1	16.2874	0.0630	0.1670
1989	6.5619	54794.6	92505.0	80220.3	97134.3	11743.1	17.6322	0.0478	0.1502
1990	6.8402	55307.1	87658.5	83670.3	105718.0	11743.1	19.4612	-0.0019	0.0532
1991	7.1797	54288.2	79231.1	91135.2	122239.5	11743.1	19.8549	0.0650	0.1449
1992	7.4056	53822.7	71961.9	92275.7	126789.8	11743.1	20.7385	0.0738	0.1873
1993	7.5141	54492.6	79527.0	91432.1	125878.0	11743.1	21.5174	0.0918	0.1744
1994	7.6295	55303.1	102483.4	89034.4	125654.0	11743.1	22.1142	0.0487	0.1522
1995	7.7065	56783.2	117724.1	91219.3	130249.1	11743.1	23.1090	0.0338	0.1611
1996	7.9883	56848.6	122736.2	94606.3	134392.0	11743.1	23.7180	0.0716	0.1645
1997				94586.6	138467.1	11743.1	25.0208		

Recall equation (13) in section 3 above which defined the declining balance capital stock in terms of vintage investments. This equation can be manipulated to yield the following relationship between the capital stock at the beginning and end of year t, K^t and K^{t+1} respectively, and investment during year t, I^t:

(A1) $K^{t+1} = (1-\delta)K^t + I^t$

where δ is the declining balance depreciation rate. We can use the information from the national accounts on investment in nonresidential structures listed in Table A1 above, Q_{NS}, along with the information from the national balance sheets on the net stock of nonresidential structures, K_{NS}, and use equation (A1) to construct implied geometric depreciation rates, δ_{NS}, that are consistent with the two data sources. Similarly, we can use the information from the national accounts on investment in nonresidential structures listed in Table A1 above, Q_{ME}, along with the information from the national balance sheets on the net stock of nonresidential structures, K_{ME}, from Table A2 and use equation (A1) to construct implied geometric depreciation rates, δ_{ME}, that are consistent with the two data sources.

Table A3: Productivity Growth Rates (%) for 18 Capital Models for Canada

Year	*g(1)*	*g(2)*	*g(3)*	*g(4)*	*g(5)*	*g(6)*	*g(7)*	*g(8)*	*g(9)*
1963	2.20	2.13	2.25	2.14	2.11	2.25	2.18	2.13	2.26
1964	3.96	3.89	3.92	3.91	3.89	3.92	3.99	3.94	3.99
1965	4.02	3.92	3.89	3.95	3.92	3.88	4.14	4.05	4.05
1966	0.99	0.84	0.92	0.85	0.81	0.90	1.16	1.02	1.12
1967	-0.59	-0.77	-0.75	-0.77	-0.79	-0.77	-0.39	-0.56	-0.53
1968	0.38	0.26	0.15	0.28	0.25	0.14	0.50	0.39	0.29
1969	0.09	-0.02	-0.01	0.03	-0.02	-0.02	0.14	0.04	0.05
1970	-2.65	-2.78	-2.66	-2.78	-2.82	-2.67	-2.60	-2.72	-2.60
1971	1.39	1.30	1.23	1.32	1.31	1.22	1.45	1.35	1.29
1972	-0.07	-0.15	-0.21	-0.18	-0.15	-0.22	-0.02	-0.10	-0.16
1973	2.14	2.06	1.97	2.01	2.01	1.94	2.19	2.10	2.02
1974	-1.19	-1.31	-1.39	-1.39	-1.39	-1.42	-1.10	-1.23	-1.31
1975	0.55	0.39	0.43	0.32	0.40	0.43	0.52	0.36	0.39
1976	1.86	1.72	1.65	1.67	1.76	1.66	1.83	1.70	1.62
1977	3.31	3.17	3.19	3.10	3.15	3.19	3.29	3.14	3.16
1978	-1.28	-1.42	-1.36	-1.44	-1.43	-1.38	-1.30	-1.45	-1.39
1979	-1.74	-1.88	-1.90	-1.96	-1.94	-1.95	-1.72	-1.86	-1.89
1980	-2.49	-2.68	-2.71	-2.82	-2.79	-2.80	-2.45	-2.64	-2.67
1981	0.48	0.30	0.01	0.24	0.16	-0.04	0.52	0.34	0.04
1982	-1.82	-2.04	-2.19	-1.85	-2.07	-2.20	-1.75	-1.98	-2.14
1983	0.37	0.30	-0.02	0.32	0.23	-0.04	0.28	0.20	-0.13
1984	3.87	3.78	3.63	3.85	3.74	3.60	3.73	3.64	3.48
1985	1.90	1.76	1.81	1.90	1.74	1.79	1.77	1.64	1.69
1986	-0.69	-0.81	-0.83	-0.72	-0.84	-0.87	-0.73	-0.84	-0.87
1987	2.34	2.24	2.20	2.20	2.15	2.11	2.31	2.22	2.18
1988	0.46	0.35	0.33	0.30	0.25	0.23	0.45	0.35	0.33
1989	-0.90	-1.01	-1.06	-1.08	-1.09	-1.14	-0.88	-0.99	-1.03
1990	-1.56	-1.65	-1.67	-1.77	-1.75	-1.77	-1.57	-1.67	-1.68
1991	-0.23	-0.28	-0.34	-0.37	-0.35	-0.40	-0.31	-0.35	-0.41
1992	0.52	0.51	0.48	0.50	0.51	0.48	0.17	0.14	0.10
1993	2.01	2.01	1.99	2.00	1.99	1.98	1.71	1.70	1.66
1994	4.33	4.32	4.32	4.33	4.32	4.32	4.06	4.04	4.04
1995	0.58	0.54	0.59	0.57	0.54	0.59	0.33	0.28	0.33
1996	0.69	0.63	0.71	0.65	0.62	0.71	0.48	0.42	0.49

The resulting depreciation rates are listed in Table A2. Although the arithmetic averages of these annual geometric depreciation rates (.0555 for nonresidential structures and .1256 for machinery and equipment) are very reasonable, it can be seen that the annual fluctuations in these rates are unacceptably large. Thus we regard the balance sheet estimates for the net capital stocks for the two reproducible capital stock components, K_{NS} and K_{ME}, as being "correct" over the entire sample period but not in the year to year movements.

Table A3: Productivity Growth Rates (%) for 18 Capital Models for Canada (cont'd)

Year	*g(10)*	*g(11)*	*g(12)*	*g(13)*	*g(14)*	*g(15)*	*g(16)*	*g(17)*	*g(18)*
1963	2.12	2.10	2.25	2.08	2.01	2.10	2.22	2.07	2.17
1964	3.95	3.94	3.99	3.90	3.84	3.85	3.90	3.83	3.59
1965	4.08	4.07	4.06	4.11	4.03	4.02	3.89	4.01	4.08
1966	1.04	1.01	1.12	1.20	1.08	1.18	0.95	1.19	0.39
1967	-0.56	-0.57	-0.54	-0.29	-0.42	-0.36	-0.69	-0.34	0.01
1968	0.41	0.38	0.28	0.55	0.45	0.38	0.20	0.40	0.86
1969	0.08	0.04	0.05	0.15	0.05	0.06	0.00	0.07	0.16
1970	-2.72	-2.76	-2.61	-2.58	-2.69	-2.57	-2.63	-2.56	-3.53
1971	1.38	1.38	1.28	1.47	1.38	1.33	1.24	1.33	1.93
1972	-0.12	-0.10	-0.17	-0.01	-0.08	-0.13	-0.20	-0.13	-0.06
1973	2.06	2.06	1.99	2.21	2.13	2.06	2.00	2.07	1.74
1974	-1.30	-1.30	-1.34	-1.03	-1.14	-1.20	-1.33	-1.16	-1.12
1975	0.29	0.37	0.39	0.54	0.39	0.43	0.51	0.52	1.48
1976	1.64	1.73	1.62	1.86	1.73	1.67	1.71	1.75	2.32
1977	3.07	3.12	3.15	3.30	3.16	3.18	3.25	3.25	4.06
1978	-1.46	-1.45	-1.40	-1.29	-1.43	-1.37	-1.34	-1.33	-1.12
1979	-1.93	-1.92	-1.94	-1.70	-1.83	-1.85	-1.87	-1.81	-1.95
1980	-2.78	-2.75	-2.77	-2.40	-2.57	-2.58	-2.66	-2.54	-2.06
1981	0.28	0.20	-0.01	0.58	0.42	0.17	0.09	0.24	1.08
1982	-1.78	-2.01	-2.14	-1.67	-1.86	-1.99	-2.09	-1.89	0.48
1983	0.23	0.14	-0.14	0.26	0.17	-0.15	0.02	-0.09	1.09
1984	3.71	3.60	3.46	3.65	3.53	3.37	3.66	3.42	3.92
1985	1.79	1.63	1.68	1.69	1.55	1.57	1.85	1.63	2.06
1986	-0.76	-0.88	-0.90	-0.77	-0.89	-0.92	-0.77	-0.86	-0.57
1987	2.19	2.13	2.10	2.27	2.17	2.13	2.29	2.20	2.65
1988	0.30	0.26	0.24	0.43	0.33	0.31	0.43	0.39	0.64
1989	-1.06	-1.06	-1.11	-0.87	-0.97	-1.02	-0.94	-0.91	-0.29
1990	-1.78	-1.77	-1.78	-1.58	-1.67	-1.68	-1.55	-1.56	-0.73
1991	-0.44	-0.42	-0.47	-0.33	-0.38	-0.45	-0.26	-0.35	0.80
1992	0.12	0.13	0.09	0.14	0.11	0.06	0.47	0.14	0.94
1993	1.69	1.68	1.66	1.68	1.66	1.62	2.02	1.70	2.01
1994	4.05	4.03	4.03	4.02	3.98	3.97	4.32	4.01	4.19
1995	0.31	0.28	0.33	0.28	0.21	0.24	0.62	0.32	0.40

1996	0.43	0.41	0.49	0.42	0.35	0.40	0.75	0.48	1.09

In Table A3, we list the TFP growth rates for each of the 18 Models discussed in the main text for the years 1963-1996.

REFERENCES

Beidelman, C. (1973), *Valuation of Used Capital Assets*, Sarasota, Florida: American Accounting Association.

Beidelman, C. (1976), "Economic Depreciation in a Capital Goods Industry", *National Tax Journal* 29, 379-390.

Bureau of Labor Statistics (1983), *Trends in Multifactor Productivity, 1948-81*, Bulletin 2178, Washington D.C.: U.S. Government Printing Office.

Christensen, L.R. and D.W. Jorgenson (1969), "The Measurement of U.S. Real Capital Input, 1929-1967", *Review of Income and Wealth* 15:4, 293-320.

Christensen, L.R. and D.W. Jorgenson (1970), "U.S. Real Product and Real Factor Input, 1929-1967", *Review of Income and Wealth* 16:1, 19-50.

Church, A. H. (1901), "The Proper Distribution of Establishment Charges, Part III", *The Engineering Magazine* 21, 904-912.

Dean, E.R. and M.J. Harper (1998), "The BLS Productivity Measurement Program", paper presented at the CRIW/NBER conference, *New Directions in Productivity Analysis*, Washington D.C., March 20-21.

Diewert, W.E. (1976), "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 115-145. (Reprinted in *Essays in Index Number Theory, Volume 1*, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North-Holland).

Diewert, W.E. (1978a), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, 883-900.

Diewert, W.E. (1978b), "Hicks' Aggregation Theorem and the Existence of a Real Value added Function", pp. 17-51 in *Production Economics: A Dual Approach to Theory and Applications, Volume 2,* M. Fuss and D. McFadden (eds.), Amsterdam: North-Holland.

(Reprinted in *Essays in Index Number Theory, Volume 1*, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North-Holland).

Diewert, W.E. (1978b), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, 883-900. (Reprinted in *Essays in Index Number Theory, Volume 1*, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North-Holland).

Diewert, W.E. (1980), Aggregation Problems in the Measurement of Capital", pp. 433-528 in *The Measurement of Capital*, D. Usher (ed.), Chicago: the University of Chicago Press.

Diewert, W.E. (1992), "The Measurement of Productivity", *Bulletin of Economic Research* 44:3, 163-198.

Epstein, L.G. and M. Denny (1980), "Endogenous Capital Utilization in a Short Run Production Model; Theory and Empirical Application", *Journal of Econometrics* 12, 189-207.

Fisher, I. (1922), *The Making of Index Numbers*, Boston: Houghton-Mifflin.

Green, J.B. (1915), "The Perpetual Inventory in Practical Stores Operation", *The Engineering Magazine* 48, 879-888.

Griliches, Z. (1963), "Capital Stock in Investment Functions: Some Problems of Concept and Measurement", in *Measurement in Economics*, C. Christ et al (eds.), Stanford: Stanford University Press, reprinted as pp. 123-143 in Z. Griliches (1988), *Technology, Education and Productivity*, New York: Basil Blackwell.

Hall, R.E. and D.W. Jorgenson (1967), "Tax Policy and Investment Behavior", *American Economic Review* 57, 391-414.

Harper, M.J., E.R. Berndt and D.O. Wood (1989), "Rates of Return and Capital Aggregation Using Alternative Rental Prices", pp. 331-372 in D.W. Jorgenson and R. Landau (eds.), *Technology and Capital Formation*, Cambridge, MA: The MIT Press.

Hicks, J.R. (1946), "*Value and Capital*", Second Edition, Oxford: Clarendon Press.

Hulten, C.R. (1990), "The Measurement of Capital", pp. 119-158 in *Fifty Years of Economic Measurement*, E.R. Berndt and J.D. Triplett (eds.), Chicago: the University of Chicago Press.

Hulten, C.R. (1996), "Capital and Wealth in the Revised SNA", pp. 149-181 in *The New System of National Accounts*, J.W. Kendrick (ed.), Boston: Kluwer Academic Publishers.

Hulten, C.R. and F.C. Wykoff (1981a), "The Estimation of Economic Depreciation Using Vintage Asset Prices: An Application of the Box-Cox Power Transformation", *Journal of Econometrics* 15, 367-396.

Hulten, C.R. and F.C. Wykoff (1981b), "The Measurement of Economic Depreciation", pp. 81-125 in *Depreciation, Inflation and the Taxation of Income from Capital*", C.R. Hulten (ed.), Washington D.C.: The Urban Institute Press.

Hulten, C.R. and F.C. Wykoff (1996), "Issues in the Measurement of Economic Depreciation Introductory Remarks", *Economic Inquiry* 34, 10-23.

Jorgenson, D.W. (1963), "Capital Theory and Investment Behavior", *American Economic Review* 53, 366-378.

Jorgenson, D.W. (1973), "The Economic Theory of Replacement and Depreciation", pp. 189-221 in *Econometrics and Economic Theory*, W. Sellekaerts (ed.), New York: Macmillan.

Jorgenson, D.W. (1989), "Capital as a Factor of Production", pp. 1-35 in *Technology and Capital Formation*, D.W. Jorgenson and R. Landau (eds.), Cambridge Mass.: MIT Press.

Jorgenson, D.W. (1996), "Empirical Studies of Depreciation", *Economic Inquiry* 34, 24-42.

Jorgenson, D.W. and Z. Griliches (1967), "The Explanation of Productivity Change", *Review of Economic Studies* 34, 249-283.

Jorgenson, D.W. and Z. Griliches (1972), "Issues in Growth Accounting: A Reply to Edward F. Denison", *Survey of Current Business* 52:5 part 2, 65-94.

Nadiri, M.I. and I.R. Prucha (1996), "Estimation of the Depreciation Rate of Physical and R&D Capital in the U.S. Total Manufacturing Sector", *Economic Inquiry* 34, 43-56.

Pakes, A. and Z. Griliches (1984), "Estimating Distributed Lags in Short Panels with an Application to the Specification of Depreciation Patterns and Capital Stock Constructs", *Review of Economic Studies* 51, 243-262.

Statistics Canada (various years), *The Input-Output Structure of the Canadian Economy 1961-1981*, Ottawa: Statistics Canada.

Statistics Canada (1984), *National Income and Expenditure Accounts, Annual Estimates, 1926-83*, Ottawa: Statistics Canada.

Statistics Canada (1989), *The Canadian Economic Observer*, May, Ottawa: Statistics Canada.

Statistics Canada (1996), *National Income and Expenditure Accounts, Annual Estimates 1984-1995*, Ottawa: Statistics Canada.

Statistics Canada (1997), *Canada's Balance of International Payments, 1926 to 1996*, Ottawa: Statistics Canada.

Triplett, J.E. (1996), "Depreciation in Production Analysis and in Income and Wealth Accounts: Resolution of an Old Debate", *Economic Inquiry* 34, 93-115.

United Nations, Eurostat, IMF, OECD and the World Bank (1993), *System of National Accounts 1993*, New York, Luxembourg, Paris and Washington D.C.

Walras, L. (1954), *Elements of Pure Economics*, translated by W. Jaffe (first published in 1874), London: George Allen and Unwin.

Wykoff, F.C. (1989), "Economic Depreciation and the User Cost of Business Leased Automobiles", pp. 259-292 in *Technology and Capital Formation*, D.W. Jorgenson and R. Landau (eds.), Cambridge, MA: The MIT Press.